

Calculus II, Section 7.6, #46  
Integration Using Tables and Computer Algebra Systems

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Computer algebra systems sometimes need a helping hand from human beings. Try to evaluate

$$\int (1 + \ln(x)) \sqrt{1 + (x \ln(x))^2} dx$$

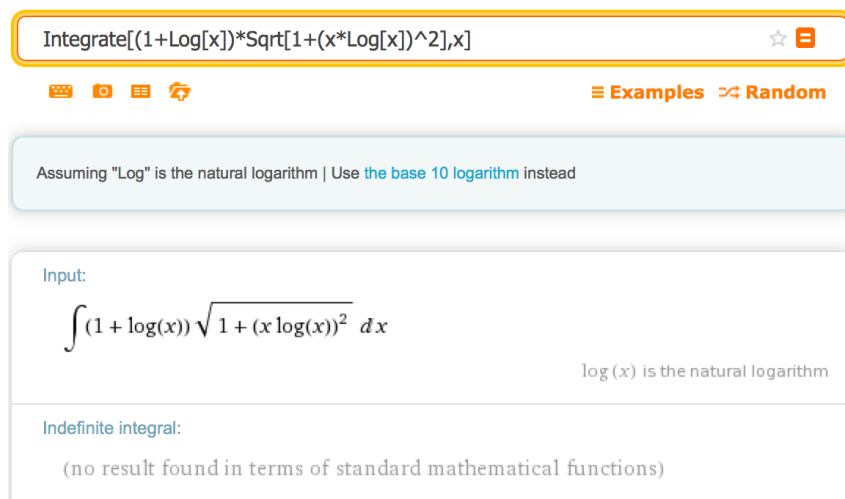
with a computer algebra system. If it doesn't return an answer, make a substitution that changes the integral into one that the CAS can integrate.<sup>1</sup>

We'll use Wolfram|Alpha (W|A).

Using Mathematica format, the input is

```
Integrate[(1+Log[x])*Sqrt[1+(x*Log[x])^2],x]
```

and we get



The screenshot shows the Wolfram|Alpha interface. At the top, the input `Integrate[(1+Log[x])*Sqrt[1+(x*Log[x])^2],x]` is entered in a search bar. Below the search bar are icons for various tools and options like "Examples" and "Random". A message states "Assuming 'Log' is the natural logarithm | Use the base 10 logarithm instead". The input is displayed as  $\int (1 + \log(x)) \sqrt{1 + (x \log(x))^2} dx$  with a note that  $\log(x)$  is the natural logarithm. The result is "Indefinite integral: (no result found in terms of standard mathematical functions)".

So W|A is unable to evaluate the integral.

Let  $u = 1 + (x \ln(x))^2$ , then  $du = 2(x \ln(x)) \left(x \cdot \frac{1}{x} + \ln(x) \cdot 1\right) = 2(x \ln(x))(1 + \ln(x)) dx$ . This is not a good result for us—the factor  $2(x \ln(x))$  is not present in the integrand—but this does show us that the derivative of  $x \ln(x)$  is present in the integrand. Let's try again.

Let  $u = x \ln(x)$ , so  $dfu = \left(x \cdot \frac{1}{x} + \ln(x) \cdot 1\right) dx = (1 + \ln(x)) dx$ .

Our integral becomes

$$\int \sqrt{1 + u^2} du$$

and W|A gives us

$$\int \sqrt{1 + u^2} du = \frac{1}{2} \left( \sqrt{u^2 + 1} u + \sinh^{-1}(u) \right) + \text{constant}$$

Computed by Wolfram|Alpha

Thus

$$\int (1 + \ln(x)) \sqrt{1 + (x \ln(x))^2} dx = \frac{1}{2} \left( x \ln(x) \sqrt{(x \ln(x))^2 + 1} + \sinh^{-1}(x \ln(x)) \right) + C$$

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<sup>1</sup>Stewart, *Calculus, Early Transcendentals*, p. 513, #46.