

Calculus II, Section 7.7, #36
Approximate Integration

Water leaked from a tank at a rate of $r(t)$ liters per hour, where the graph of r is as shown. Use Simpson's Rule to estimate the total amount of water that leaked out in the first six hours.¹



We are given a graph of the rate of water leakage as a function of time, so the area under that rate graph will give us the amount of water that leaks from the tank in the first six hours.

We will be able to most accurately estimate the values of $r(t)$ using the vertical grid lines, so we'll take our $\Delta t = 1$.

Let's make a table of our estimates of $r(t)$:

t	0	1	2	3	4	5	6
$r(t)$	4	3	2.3	1.9	1.4	1.1	1

For this data, Simpson's Rule tells us

$$\begin{aligned}\int_0^6 r(t) dt &\approx S_6 = \frac{\Delta t}{3} [r(0) + 4 \cdot r(1) + 2 \cdot r(2) + 4 \cdot r(3) + 2 \cdot r(4) + 4 \cdot r(5) + r(6)] \\ &= \frac{1}{3} [4 + 4 \cdot 3 + 2 \cdot 2.3 + 4 \cdot 1.9 + 2 \cdot 1.4 + 4 \cdot 1.1 + 1] \\ &\approx 12.13\end{aligned}$$

Thus in the first six hours, about 12.13 liters of water leaked from the tank.

¹Stewart, *Calculus, Early Transcendentals*, p. 526, #36.