

Calculus II, Section 7.7, #40
Approximate Integration

The table shows the values of a force function $f(x)$, where x is measured in meters and $f(x)$ in newtons. Use Simpson's Rule to estimate the work done by the force in moving an object a distance of 18 m.¹

x	0	3	6	9	12	15	18
$f(x)$	9.8	9.1	8.5	8.0	7.7	7.5	7.4

For this data, we have $\Delta x = 3$, so Simpson's Rule tells us

$$\begin{aligned}\text{Work} &= \int_0^{18} f(x) \, dx \approx S_6 = \frac{\Delta t}{3} [f(0) + 4 \cdot f(1) + 2 \cdot f(2) + 4 \cdot f(3) + 2 \cdot f(4) + 4 \cdot f(5) + f(6)] \\ &= 1 \cdot [9.8 + 4 \cdot 9.1 + 2 \cdot 8.5 + 4 \cdot 8.0 + 2 \cdot 7.7 + 4 \cdot 7.5 + 7.4] \\ &= 148\end{aligned}$$

Thus over the distance of 18 m, the amount of work done is 148 J.

¹Stewart, *Calculus, Early Transcendentals*, p. 526, #40.