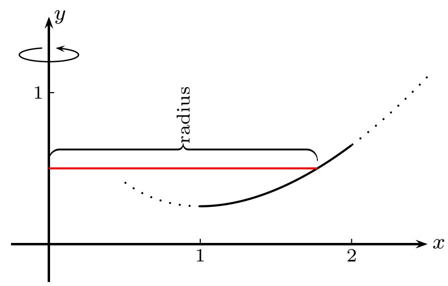
The given curve is rotated about the y-axis. Find the area of the resulting surface.[1]

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x) \quad 1 \le x \le 2$$

The diagram at right shows the curve being revolved about the y-axis, along with a radius.



Since

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$$

we have

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2}x - \frac{1}{2x} \\ &= \frac{x^2}{2x} - \frac{1}{2x} \\ &= \frac{x^2-1}{2x}\end{aligned}$$

So the arc length differential ds is

$$ds = \sqrt{1 + \left(\frac{x^2-1}{2x}\right)^2}\, dx = \sqrt{1 + \frac{(x^2-1)^2}{4x^2}}\, dx = \sqrt{\frac{4x^2 + x^4 - 2x^2 + 1}{4x^2}}\, dx$$

and since $x \ge 0$,

$$ds = \frac{\sqrt{x^4 + 2x^2 + 1}}{2x}\, dx = \frac{\sqrt{(x^2+1)^2}}{2x}\, dx$$

and since $x^2 + 1 \ge 0$

$$ds = \frac{x^2+1}{2x}\, dx$$

In terms of x, the radius is x, the arc length differential is $\frac{x^2+1}{2x}\, dx$, and we integrate from $x = 1$ to $x = 2$. We get

$$\begin{aligned}S &= \int_{x=1}^{x=2} 2\pi x \cdot \frac{x^2+1}{2x}\, dx \\ &= \pi \int_{x=1}^{x=2} x^2 + 1\, dx \\ &= \pi \left[\frac{x^3}{3} + x\right]_{x=1}^{x=2} \\ &= \pi \left[\left(\frac{2^3}{3} + 2\right) - \left(\frac{1^3}{3} + 1\right) \right] \\ &= \pi \left[\frac{7}{3} + 1\right] \\ &= \frac{10\pi}{3}\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 556, #18.