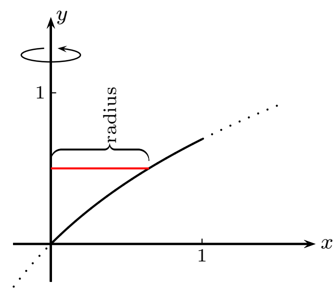
Calculus II, Section 8.2, #26
Area of a Surface of Revolution

Use a CAS (Computer Algebra System) to find the exact area of the surface obtained by rotating the curve about the y-axis. If your CAS has trouble evaluating the integral, express the surface area as an integral in the other variable.[1]

$$y = \ln(x+1), \quad 0 \le x \le 1$$

The diagram at right shows the curve being revolved about the y-axis, along with a radius.



Since

$$y = \ln(x+1)$$

we have

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x+1}$$

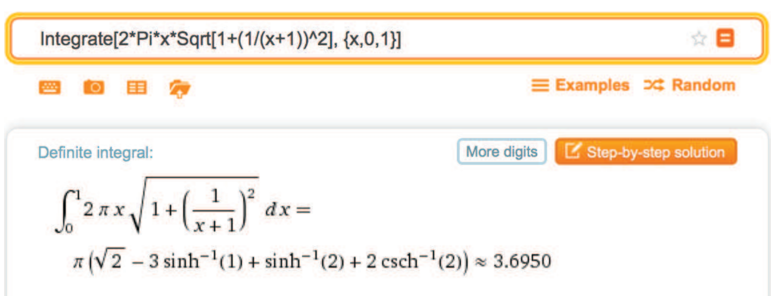
So the arc length differential ds is

$$\mathrm{d}s = \sqrt{1 + \left(\frac{1}{x+1}\right)^2}\, \mathrm{d}x$$

In terms of x, the radius is x, the arc length differential is $\sqrt{1 + \left(\frac{1}{x+1}\right)^2}\, \mathrm{d}x$, and we integrate from $x = 0$ to $x = 1$. We get

$$S = \int_{x=0}^{x=1} 2\pi x \cdot \sqrt{1 + \left(\frac{1}{x+1}\right)^2}\, \mathrm{d}x$$

WolframAlpha gives



In terms of y, the radius is $\mathrm{e}^y - 1$, $\frac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^y$, so the arc length differential is

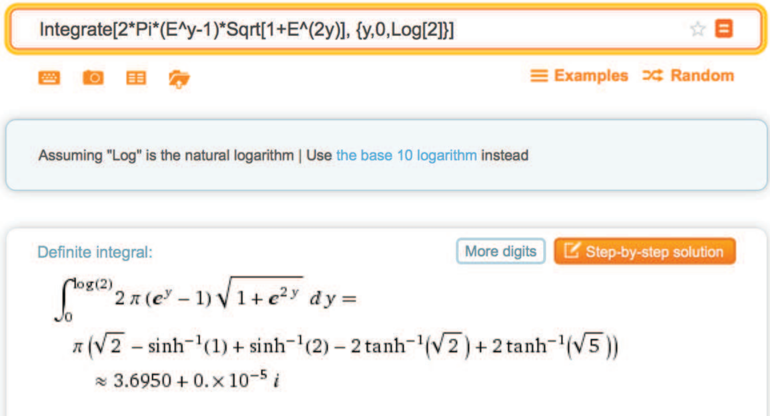
$$\mathrm{d}s = \sqrt{1 + (\mathrm{e}^y)^2}\, \mathrm{d}y$$
$$= \sqrt{1 + \mathrm{e}^{2y}}\, \mathrm{d}y$$

and we integrate from $y = 0$ to $y = \ln(2)$. We get

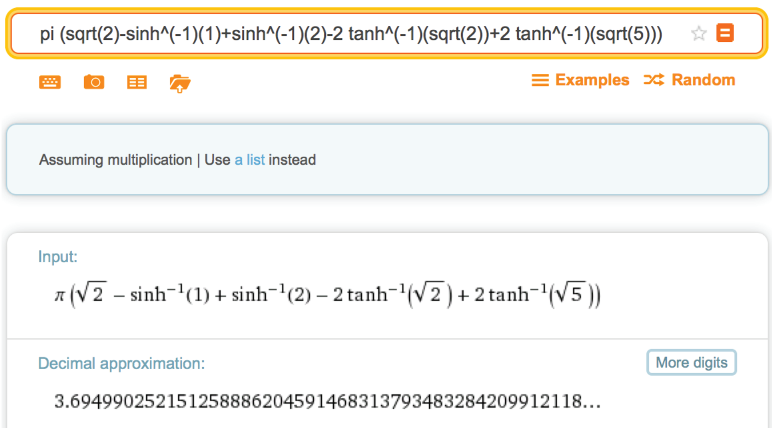
$$S = \int_{y=0}^{y=\ln(2)} 2\pi\left(\mathrm{e}^y - 1\right)\sqrt{1 + \mathrm{e}^{2y}}\, \mathrm{d}y$$

WolframAlpha gives

[1] Stewart, *Calculus, Early Transcendentals*, p. 556, #26.



which is an interesting result because it contains a complex portion. If we ask WolframAlpha to compute the exact portion, we get



The approximations for the integral with respect to either x or y are both ≈ 3.6950, so the exact area can be expressed in either way.