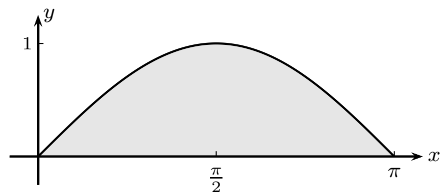
Sketch the region bounded by the curves, and visually estimate the location of the centroid. Then find the exact coordinates of the centroid.[1]

$$y = \sin(x), \quad y = 0, \quad 0 \le x \le \pi$$

Here's a sketch of the region:



From the symmetry about the line $x = \frac{\pi}{2}$, it seems that the x-coordinate of the centroid will be $\frac{\pi}{2}$. I'd visually estimate the y-coordinate to be ≈ 0.35.

The relevant formulas for the centroid (center of mass of the region) are

$$\bar{x} = \frac{1}{A}\int_a^b x\left[f(x) - g(x)\right]\,\mathrm{d}x, \qquad \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}\left\{[f(x)]^2 - [g(x)]^2\right\}\,\mathrm{d}x$$

For this problem, $a = 0$, $b = \pi$, $f(x) = sin(x)$, and $g(x) = 0$.

The area, A, is given by

$$\begin{aligned} A &= \int_0^\pi \sin(x) - 0\,\mathrm{d}x \\ &= [-\cos(x)]_0^\pi \\ &= -\cos(\pi) - -\cos(0) \\ &= -(-1) - -(1) \\ &= 2 \end{aligned}$$

So

$$\begin{aligned} \bar{x} &= \frac{1}{2}\int_0^\pi x\left[\sin(x) - 0\right]\,\mathrm{d}x \\ &= \frac{1}{2}\int_0^\pi x\sin(x)\,\mathrm{d}x \\ &= \frac{1}{2}\left[-x\cos(x) + \sin(x)\right]_0^\pi \\ &= \frac{1}{2}\left[(-\pi\cdot\cos(\pi) + \sin(\pi)) - (-0\cdot\cos(0) + \sin(0))\right] \\ &= \frac{1}{2}[\pi] \\ &= \frac{\pi}{2} \end{aligned}$$

Integration by parts:

$$\int x\sin(x)\,\mathrm{d}x$$

Let $u = x$, so $\mathrm{d}v = \sin(x)\mathrm{d}x$. Then $\mathrm{d}u = \mathrm{d}x$ and $v = -\cos(x)$.

$$\begin{aligned} &= x\cdot -\cos(x) - \int -\cos(x)\,\mathrm{d}x \\ &= -x\cos(x) + \int \cos(x)\,\mathrm{d}x \\ &= -x\cos(x) + \sin(x) \end{aligned}$$

and

$$\bar{y} = \frac{1}{2}\int_0^\pi \frac{1}{2}\left[\sin^2(x) - 0^2\right]\,\mathrm{d}x$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 567, #28.

$$= \frac{1}{2}\int_0^{\pi} \frac{1}{2}\sin^2(x)\,\mathrm{d}x$$

Using the trigonometric identity $\sin^2(x) = \frac{1-\cos(2x)}{2} = \frac{1}{2}(1-\cos(2x))$,

$$\begin{aligned}
&= \frac{1}{2}\int_0^{\pi} \frac{1}{2}\cdot\frac{1}{2}(1-\cos(2x))\,\mathrm{d}x \\
&= \frac{1}{8}\int_0^{\pi} 1-\cos(2x)\,\mathrm{d}x \\
&= \frac{1}{8}\left[x - \frac{\sin(2x)}{2}\right]_0^{\pi} \\
&= \frac{1}{8}\left[\left(\pi - \frac{\sin(2\pi)}{2}\right) - \left(0 - \frac{\sin(0)}{2}\right)\right] \\
&= \frac{1}{8}[\pi - 0 - 0 + 0] \\
&= \frac{\pi}{8} \\
&= 0.3927
\end{aligned}$$

Thus, the exact coordinates of the centroid are $\left(\frac{\pi}{2}, \frac{\pi}{8}\right)$ or about (1.5708,0.3927).