

High blood pressure results from constriction of the arteries. To maintain a normal flow rate (flux), the heart has to pump harder, thus increasing the blood pressure.¹ Use Poiseuille's Law to show that if R_0 and P_0 are normal values of the radius and pressure in an artery and the constricted values are R and P then for the flux to remain constant, P and R are related by the equation

$$\frac{P}{P_0} = \left(\frac{R_0}{R}\right)^4$$

Deduce that if the radius of an artery is reduced to three-fourths of its former value, then the pressure is more than tripled.²

Poiseuille's Law is

$$F = \frac{\pi P R^4}{8\eta l}$$

where F is the flux, P is the pressure, R is the radius, η is the viscosity of the blood, and l is the length of the artery.

If we substitute the normal values, then for the flux to remain constant, we must have

$$\begin{aligned}\frac{\pi P_0 R_0^4}{8\eta l} &= \frac{\pi P R^4}{8\eta l} \\ P_0 R_0^4 &= P R^4\end{aligned}$$

or

$$\begin{aligned}\frac{R_0^4}{R^4} &= \frac{P}{P_0} \\ \frac{P}{P_0} &= \left(\frac{R_0}{R}\right)^4\end{aligned}$$

Now, if we substitute $\frac{3}{4}R_0$ for R , and solve for P , we get

$$\begin{aligned}\frac{P}{P_0} &= \left(\frac{R_0}{\frac{3}{4}R_0}\right)^4 \\ P &= P_0 \cdot \left(\frac{4}{3}\right)^4 \\ P &\approx P_0 \cdot 3.1605\end{aligned}$$

Thus if the radius of the artery is reduced to three-fourths of its former value, then the pressure is more than tripled.³

¹Eat your veggies and exercise regularly to avoid this killer condition.

²Stewart, *Calculus, Early Transcendentals*, p. 573, #20.

³Did I mention . . . eat your veggies, exercise regularly, and reduce your fat intake. Your clothes will fit better and your heart will be happy to pump for a long time.