The time between infection and display of symptoms for streptococcal sore throat is a random variable whose probability density function can be approximated by $f(t) = \frac{1}{15{,}676}t^2 e^{-0.05t}$ *if* $0 \le t \le 150$ *and* $f(t) = 0$ *otherwise (t measured in hours).*[1]

(a) *What is the probability that an infected patient will display symptoms within the first* 48 hours*?*

Since we know the probability density function, we have

$$\begin{aligned} P(0 \le T \le 48) &= \int_{t=0}^{t=48} \frac{1}{15{,}676} t^2 e^{-0.05t}\, dt \\ &= \frac{1}{15{,}676} \cdot \int_{t=0}^{t=48} t^2 e^{-0.05t}\, dt \end{aligned}$$

Using integration by parts[2], we find $\int t^2 e^{-0.05t} dt$ and then use the FTC to evaluate the definite integral.

$$\begin{aligned} &= \frac{1}{15{,}676} \cdot \left[-20e^{-0.05t}\left(t^2 + 40t + 800\right)\right]_{t=0}^{t=48} \\ &= \frac{1}{15{,}676} \cdot \left[\left(-20e^{-0.05\cdot 48}\left(48^2 + 40\cdot 48 + 800\right)\right) - \left(-20e^{-0.05\cdot 0}\left(0^2 + 40\cdot 0 + 800\right)\right)\right] \\ &\approx 0.4392 \end{aligned}$$

Thus, there is a 43.92% probability that an infected patient will display symptoms within the first 48 hours.

(b) *What is the probability that an infected patient will not display symptoms until after* 36 hours*?*

We want

$$\begin{aligned} P(T > 36) &= P(36 \le T \le 150) \\ &= \int_{t=36}^{t=150} \frac{1}{15{,}676} t^2 e^{-0.05t}\, dt \\ &= \frac{1}{15{,}676} \cdot \int_{t=36}^{t=150} t^2 e^{-0.05t}\, dt \\ &= \frac{1}{15{,}676} \cdot \left[-20e^{-0.05t}\left(t^2 + 40t + 800\right)\right]_{t=36}^{t=150} \\ &= \frac{1}{15{,}676} \cdot \left[\left(-20e^{-0.05\cdot 150}\left(150^2 + 40\cdot 150 + 800\right)\right) - \left(-20e^{-0.05\cdot 36}\left(36^2 + 40\cdot 36 + 800\right)\right)\right] \\ &\approx 0.7250 \end{aligned}$$

Thus, there is a 72.50% probability that an infected patient will display symptoms after 36 hours.

[1] Stewart, *Calculus, Early Transcendentals*, p. 580, #12.
[2] Details at the end of the problem.

Let's compute

$$\int t^2 \mathrm{e}^{-0.05t}\,\mathrm{d}t$$

using tabular integration by parts.

Here's the table

u		$\mathrm{d}v$
t^2		$\mathrm{e}^{-0.05t}$
$2t$	$+$	$-\frac{1}{0.05}\mathrm{e}^{-0.05t}$
2	$-$	$\frac{1}{(0.05)^2}\mathrm{e}^{-0.05t}$
0	$+$	$-\frac{1}{(0.05)^3}\mathrm{e}^{-0.05t}$

Thus,

$$\begin{aligned}
\int t^2 \mathrm{e}^{-0.05t}\,\mathrm{d}t &= +t^2 \cdot -\frac{1}{0.05}\mathrm{e}^{-0.05t} - 2t \cdot \frac{1}{(0.05)^2}\mathrm{e}^{-0.05t} + 2 \cdot -\frac{1}{(0.05)^3}\mathrm{e}^{-0.05t} - \int 0 \cdot -\frac{1}{(0.05)^3}\mathrm{e}^{-0.05t}\,\mathrm{d}t \\
&= t^2 \cdot -20\mathrm{e}^{-0.05t} - 2t \cdot 400\mathrm{e}^{-0.05t} + 2 \cdot -8000\mathrm{e}^{-0.05t} - \int 0\,\mathrm{d}t \\
&= -20t^2\mathrm{e}^{-0.05t} - 800t\mathrm{e}^{-0.05t} - 16000\mathrm{e}^{-0.05t} + C \\
&= -20\mathrm{e}^{-0.05t}\left(t^2 + 400t + 800\right) + C
\end{aligned}$$

Since we'll be using this antiderivative in a definite integral, we can ignore the "$+C$". (When used in the FTC calculation of a definite integral, the C always subtracts out.)