

Calculus II, Section 9.2, #24
Direction Fields and Euler's Method

- (a) Use Euler's method with step size 0.2 to estimate $y(0.6)$, where $y(x)$ is the solution of the initial value problem $y' = \cos(x + y)$, $y(0) = 0$.¹

A table will help to organize our computations.

n	$x_n = x_{n-1} + h$	$y_n = y_{n-1} + h \cdot F(x_{n-1}, y_{n-1})$
0	0	0
1	$0 + 0.2 = 0.2$	$0 + 0.2 \cdot \cos(0 + 0) = 0.2$
2	$0.2 + 0.2 = 0.4$	$0.2 + 0.2 \cdot \cos(0.2 + 0.2) \approx 0.38421220$
3	$0.4 + 0.2 = 0.6$	$0.38421220 + 0.2 \cdot \cos(0.4 + 0.38421220) \approx 0.52580118$

Thus, $y(0.6) \approx 0.5258$.

- (b) Repeat part (a) with step size 0.1.

A table will help to organize our computations, but a spreadsheet would be the best choice for this computation.

n	$x_n = x_{n-1} + h$	$y_n = y_{n-1} + h \cdot F(x_{n-1}, y_{n-1})$
0	0	0
1	$0 + 0.1 = 0.1$	$0 + 0.1 \cdot \cos(0 + 0) = 0.1$
2	$0.1 + 0.1 = 0.2$	$0.1 + 0.1 \cdot \cos(0.1 + 0.1) \approx 0.19800666$
3	$0.2 + 0.1 = 0.3$	$0.19800666 + 0.1 \cdot \cos(0.2 + 0.19800666) \approx 0.29019020$
4	$0.3 + 0.1 = 0.4$	$0.29019020 + 0.1 \cdot \cos(0.3 + 0.29019020) \approx 0.37327368$
5	$0.4 + 0.1 = 0.5$	$0.37327368 + 0.1 \cdot \cos(0.4 + 0.37327368) \approx 0.44483647$
6	$0.5 + 0.1 = 0.6$	$0.44483647 + 0.1 \cdot \cos(0.5 + 0.44483647) \approx 0.50342401$

Thus, $y(0.6) \approx 0.5034$.

¹Stewart, *Calculus, Early Transcendentals*, p. 599, #24.