

- (a) Use a computer algebra system (CAS) to draw a direction field for the differential equation. Get a printout and use it to sketch some solution curves without solving the differential equation.¹

$$y' = xy$$

We used `dfield`, a Java application found at <http://math.rice.edu/~dfield/dfpp.html>. It took a couple of minutes to figure out how to use the application and the settings we ended up with are shown in Figure 1. The direction field is shown in Figure 2. Figure 3 shows the direction field with three particular solutions.

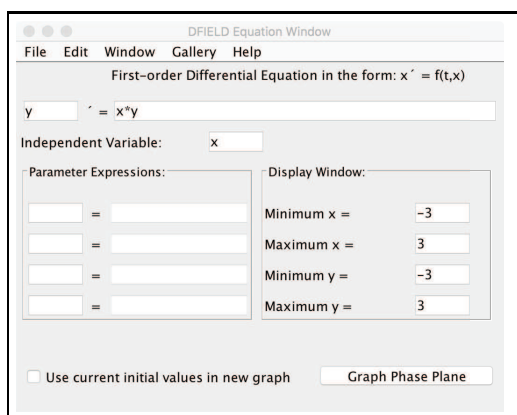


Figure 1: `dfield` Settings for $y' = xy$.

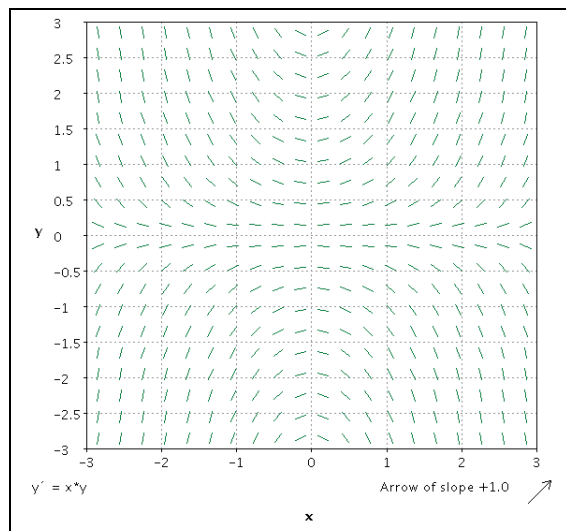


Figure 2: Direction field for $y' = xy$.

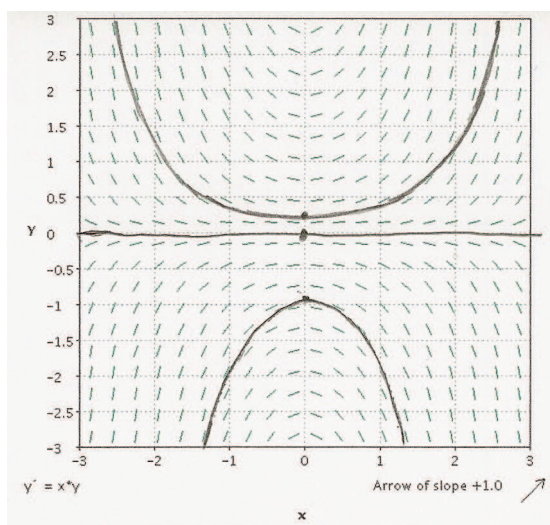


Figure 3: Direction field with particular solutions.

¹Stewart, *Calculus, Early Transcendentals*, p. 605, #28.

Calculus II
 Separable Equations

(b) Solve the differential equation.

We separate x and y , and integrate.

$$\begin{aligned} \frac{dy}{dx} &= xy \\ \frac{1}{y} dy &= x dx \\ \int \frac{1}{y} dy &= \int x dx \\ \ln |y| &= \frac{1}{2}x^2 + C_1 \\ |y| &= e^{x^2/2+C_1} \\ |y| &= e^{x^2/2} \cdot e^{C_1} \\ |y| &= e^{x^2/2} \cdot C_2 \end{aligned}$$

Since e^{C_1} is just a constant, we write

$$\begin{aligned} |y| &= C_2 e^{x^2/2} \\ y &= C_2 e^{x^2/2} \quad \text{or} \quad -y = C_2 e^{x^2/2} \\ y &= C_2 e^{x^2/2} \quad \text{or} \quad y = -C_2 e^{x^2/2} \end{aligned}$$

Since C_2 and $-C_2$ are just constants, we write

$$y = C e^{x^2/2}$$

(c) Use the CAS to draw several members of the family of solutions obtained in part (b). Compare with the curves from part (a).

We entered the initial conditions for the particular solutions by accessing the Solution menu in the DFIELD Direction Field Window. Figure 4 shows one set of initial conditions, and Figure 5 shows the particular solutions for $y(0) = 0.2$, $y(0) = 0$, and $y(0) = -1$.

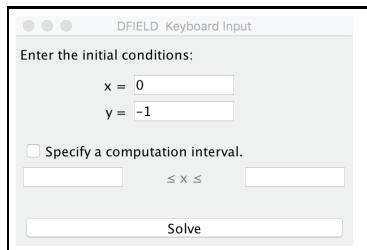


Figure 4: dfield Settings for $y' = xy$.

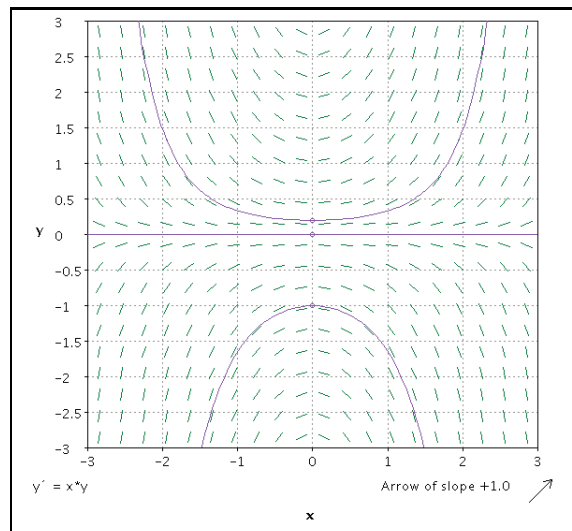


Figure 5: Direction field for $y' = xy$.