Find the orthogonal trajectories of the family of curves. Use a graphing device to draw several members of each family on a common screen.[1]

$$y^2 = kx^3$$

First, we find the slopes of the given family.

$$\begin{aligned}
y^2 &= kx^3 \\
2y \,\mathrm{d}y &= 3kx^2 \,\mathrm{d}x \\
\frac{1}{2y\,\mathrm{d}x} \cdot \mathrm{d}y &= \frac{1}{2y\,\mathrm{d}x} \cdot 3kx^2 \,\mathrm{d}x \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{3kx^2}{2y}
\end{aligned}$$

From $y^2 = kx^3$ we get $k = \frac{y^2}{x^3}$. Substituting,

$$\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{3 \cdot \left(\frac{y^2}{x^3}\right) \cdot x^2}{2y} \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{3y}{2x}
\end{aligned}$$

This equation gives the slope of the original family at any value (x,y). Then the slope of the orthogonal (perpendicular) line must be

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x}{3y}$$

and we solve this differential equation to get the equation for the family of orthogonal trajectories.

$$\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{2x}{3y} \\
3y \,\mathrm{d}y &= -2x \,\mathrm{d}x \\
\int 3y \,\mathrm{d}y &= \int -2x \,\mathrm{d}x \\
\frac{3}{2}y^2 &= -x^2 + C_1 \\
\frac{2}{3} \cdot \frac{3}{2}y^2 &= \frac{2}{3} \cdot \left(-x^2 + C_1\right) \\
y^2 &= -\frac{2}{3}x^2 + \frac{2}{3}C_1 \\
\frac{2}{3}x^2 + y^2 &= k \qquad \text{where } k = \frac{2}{3}C_1 \text{ is a constant.} \\
\frac{x^2}{\frac{3k}{2}} + \frac{y^2}{k} &= 1 \\
\frac{x^2}{\left(\sqrt{\frac{3k}{2}}\right)^2} + \frac{y^2}{\left(\sqrt{k}\right)^2} &= 1
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 605, #30.

Thus the family of orthogonal trajectories for $y^2 = kx^3$ is a family of ellipses of the form

$$\frac{x^2}{\left(\sqrt{\frac{3k}{2}}\right)^2} + \frac{y^2}{\left(\sqrt{k}\right)^2} = 1$$