Find an equation of the tangent line to the curve at the point corresponding to the given value of the parameter.[1]

$$x = \mathrm{e}^t \sin(\pi t), \qquad y = \mathrm{e}^{2t}; \qquad t = 0$$

To write the equation of the tangent line (indeed, any line) we need the slope of the line and a point on the line.

The point on the line is given by

$$\begin{aligned} x(0) &= \mathrm{e}^0 \sin(\pi \cdot 0) \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} y(0) &= \mathrm{e}^{2\cdot 0} \\ &= 1 \end{aligned}$$

So the point on the line is (0,1).

To find the slope, we compute the derivative, and then evaluate at $t = 0$.

$$x = \mathrm{e}^t \sin(\pi t) \qquad\qquad y = \mathrm{e}^{2t}$$

so

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \mathrm{e}^t \cdot \cos(\pi t) \cdot \pi + \sin(\pi t) \cdot \mathrm{e}^t \\ &= \mathrm{e}^t \left(\pi \cos(\pi t) + \sin(\pi t)\right) \end{aligned} \qquad\qquad \begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}t} &= \mathrm{e}^{2t} \cdot 2 \\ &= 2\mathrm{e}^{2t} \end{aligned}$$

We have

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} \\ &= \frac{2\mathrm{e}^{2t}}{\mathrm{e}^t \left(\pi \cos(\pi t) + \sin(\pi t)\right)} \\ &= \frac{2\mathrm{e}^t}{\pi \cos(\pi t) + \sin(\pi t)} \end{aligned}$$

so

$$\begin{aligned} \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=0} &= \frac{2\mathrm{e}^0}{\pi \cos(\pi \cdot 0) + \sin(\pi \cdot 0)} \\ &= \frac{2}{\pi} \end{aligned}$$

Using point-slope form for the equation of a line, we get

$$y - 1 = \frac{2}{\pi}(x - 0)$$

or

$$y = \frac{2}{\pi} x + 1$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 655, #6.