Graoh the curve in a viewing rectangle that displays all the important aspects of the curve.[1]

$$x = t^4 + 4t^3 - 8t^2, \qquad y = 2t^2 - t$$

We know

$$x = t^4 + 4t^3 - 8t^2 \qquad\qquad y = 2t^2 - t$$

so

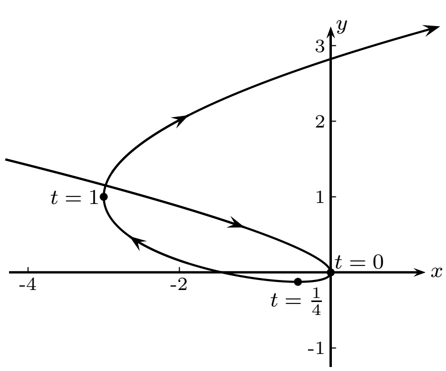
$$\frac{dx}{dt} = 4t^3 + 12t^2 - 16t \qquad\qquad \frac{dy}{dt} = 4t - 1$$
$$\frac{dx}{dt} = 4t\,(t+4)\,(t-1) \qquad\qquad \frac{dy}{dt} = 4t - 1$$

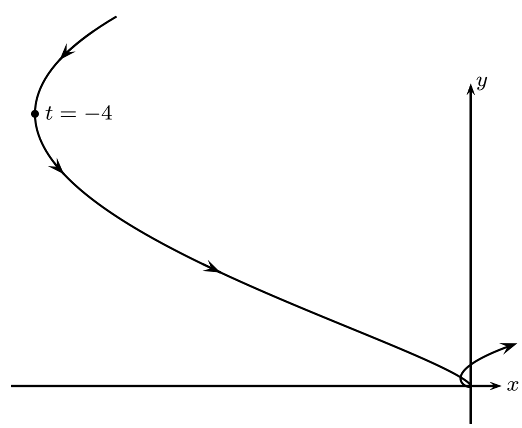
thus

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t-1}{4t\,(t+4)\,(t-1)}$$

Since $\frac{dy}{dt} = 0$ when $4t - 1 = 0 \Rightarrow t = \frac{1}{4}$ and $\frac{dx}{dt} \neq 0$ for these values of t, there is a horizontal tangent line at $t = \frac{1}{4}$ or $y = 2\left(\frac{1}{4}\right)^2 - \frac{1}{4} = -\frac{1}{8}$.

Since $\frac{dx}{dt} = 0$ when $t = -4$, 0, and 1 and $\frac{dy}{dt} \neq 0$ for these values of t, the curve has vertical tangent lines at $t = -4$, 0, and 1 or, $x = -128$, $x = 0$, and $x = -3$, respectively





[1] Stewart, *Calculus, Early Transcendentals*, p. 655, #24.