Calculus II, Section 10.3, #14
Polar Coordinates

Find a formula for the distance between the points with polar coordinates (r_1,θ_1) *and* (r_2,θ_2).[1]

Converting the point (r_1,θ_1) into rectangular coordinates gives us

$$(r_1\cos(\theta_1),r_1\sin(\theta_1))$$

and the point (r_2,θ_2) gives us

$$(r_2\cos(\theta_2),r_2\sin(\theta_2))$$

Applying the distance formula to these points in rectangular coordiantes gives

$$\begin{aligned}
d &= \sqrt{(r_2\cos(\theta_2)-r_1\cos(\theta_1))^2+(r_2\sin(\theta_2)-r_1\sin(\theta_1))^2} \\
&= \sqrt{r_2^2\cos^2(\theta_2)-2r_1r_2\cos(\theta_2)\cos(\theta_1)+r_1^2\cos^2(\theta_1)+r_2^2\sin^2(\theta_2)-2r_1r_2\sin(\theta_2)\sin(\theta_1)+r_1^2\sin^2(\theta_1)} \\
&= \sqrt{r_2^2\left(\cos^2(\theta_2)+\sin^2(\theta_2)\right)-2r_1r_2\left(\cos(\theta_1)\cos(\theta_2)+\sin(\theta_1)\sin(\theta_2)\right)+r_1^2\left(\cos^2(\theta_1)+\sin^2(\theta_1)\right)} \\
&= \sqrt{r_2^2+r_1^2-2r_1r_2\left(\cos(\theta_1)\cos(\theta_2)+\sin(\theta_1)\sin(\theta_2)\right)} \\
&= \sqrt{r_2^2+r_1^2-2r_1r_2\left(\cos(\theta_1-\theta_2)\right)}
\end{aligned}$$

Thus, the distance d between the points with polar coordinates (r_1,θ_1) and (r_2,θ_2) is given by

$$d=\sqrt{r_2^2+r_1^2-2r_1r_2\left(\cos(\theta_1-\theta_2)\right)}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 666, #14.