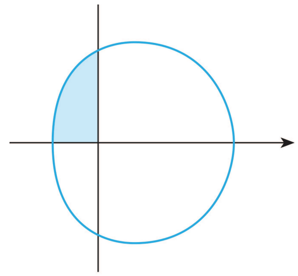
Find the area of the shaded region.[1]



$r = 2 + \cos\theta$

A quick investigation with the graphing calculator shows that the region is bounded by $\frac{\pi}{2} \le \theta \le \pi$.

We know

$$A = \int_{\theta=\alpha}^{\theta=\beta} \frac{1}{2} r^2 \, \mathrm{d}\theta$$

so

$$\begin{aligned}
A &= \int_{\theta=\pi/2}^{\theta=\pi} \frac{1}{2} \left(2+\cos(\theta)\right)^2 \, \mathrm{d}\theta \\
&= \frac{1}{2} \int_{\theta=\pi/2}^{\theta=\pi} 4 + 4\cos(\theta) + \cos^2(\theta) \, \mathrm{d}\theta \\
&= \frac{1}{2} \int_{\theta=\pi/2}^{\theta=\pi} 4 \, \mathrm{d}\theta + \frac{1}{2} \int_{\theta=\pi/2}^{\theta=\pi} 4\cos(\theta) \, \mathrm{d}\theta + \frac{1}{2} \int_{\theta=\pi/2}^{\theta=\pi} \cos^2(\theta) \, \mathrm{d}\theta \\
&= \frac{1}{2} \Big[4\theta\Big]_{\theta=\pi/2}^{\theta=\pi} + \frac{1}{2} \Big[4\cos(\theta)\Big]_{\theta=\pi/2}^{\theta=\pi} + \frac{1}{2} \int_{\theta=\pi/2}^{\theta=\pi} \frac{1}{2} \left(1+\cos(2\theta)\right) \, \mathrm{d}\theta \\
&= \frac{1}{2}\left(4\cdot\pi - 4\cdot\frac{\pi}{2}\right) + \frac{1}{2}\left(4\cos(\pi) - 4\cos\left(\frac{\pi}{2}\right)\right) + \frac{1}{4}\left[\theta + \frac{1}{2}\sin(2\theta)\right]_{\theta=\pi/2}^{\theta=\pi} \\
&= \pi - 2 + \frac{1}{4}\left(\left(\pi + \frac{1}{2}\sin(2\cdot\pi)\right) - \left(\frac{\pi}{2} + \frac{1}{2}\sin\left(2\cdot\frac{\pi}{2}\right)\right)\right) \\
&= \pi - 2 + \frac{1}{4}\left((\pi+0) - \left(\frac{\pi}{2}+0\right)\right) \\
&= \pi - 2 + \frac{1}{4}\cdot\frac{\pi}{2} \\
&= \frac{9\pi}{8} - 2 \\
&= \frac{9\pi - 16}{2}
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 672, #6.