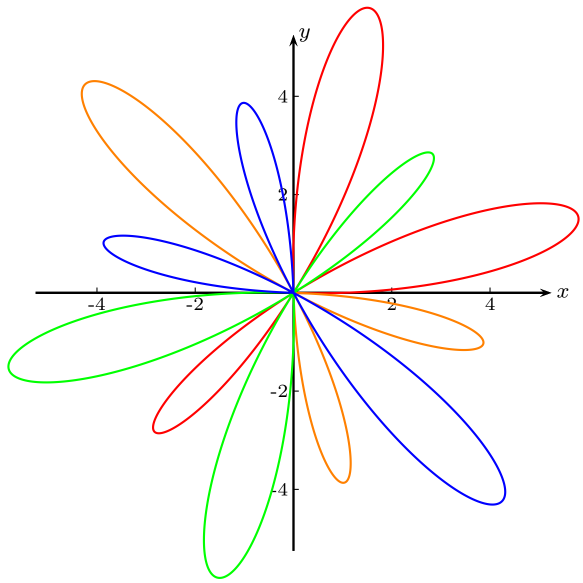
Graph the curve and find the area that it encloses.[1]

$$r = 1 + 5\sin(6\theta)$$

The curve is graphed for $0 \le \theta \le 2\pi$. (Approximately $-0.0336 \le \theta \le 1.6044$, $1.6044 \le \theta \le 3.1080$, $3.1080 \le \theta \le 4.7459$, $4.7459 \le \theta \le 6.2496$)



We know

$$A = \int_{\theta=\alpha}^{\theta=\beta} \frac{1}{2} r^2 \, \mathrm{d}\theta$$

so

$$\begin{aligned}
A &= \int_{\theta=0}^{\theta=2\pi} \frac{1}{2} \left(1 + 5\cos(6\theta)\right)^2 \, \mathrm{d}\theta \\
&= \frac{1}{2} \int_{\theta=0}^{\theta=2\pi} 1 + 10\sin(6\theta) + 25\sin^2(6\theta) \, \mathrm{d}\theta \\
&= \frac{1}{2} \int_{\theta=0}^{\theta=2\pi} 1 + 10\sin(6\theta) + 25 \cdot \frac{1}{2}\left(1 - \cos(12\theta)\right) \, \mathrm{d}\theta \\
&= \frac{1}{2} \int_{\theta=0}^{\theta=2\pi} 1 + 10\sin(6\theta) + \frac{25}{2}\left(1 - \cos(12\theta)\right) \, \mathrm{d}\theta \\
&= \frac{1}{2}\left[\theta + 10 \cdot \frac{1}{6} \cdot -\cos(6\theta) + \frac{25}{2}\theta - \frac{25}{2} \cdot \frac{1}{12}\sin(12\theta)\right]_{\theta=0}^{\theta=2\pi} \\
&= \frac{1}{2}\left[\frac{27}{2}\theta - \frac{5}{3}\cos(6\theta) - \frac{25}{24}\sin(12\theta)\right]_{\theta=0}^{\theta=2\pi} \\
&= \frac{1}{2}\left[\left(\frac{27}{2} \cdot 2\pi - \frac{5}{3}\cos(12\pi) - \frac{25}{24}\sin(24\pi)\right) - \left(0 - \frac{5}{3} - 0\right)\right] \\
&= \frac{1}{2}\left[27\pi - \frac{5}{3} - 0 - 0 + \frac{5}{3} + 0\right] \\
&= \frac{27}{2}\pi
\end{aligned}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 672, #16.