

Calculus II, Section 11.1, #30
Sequences

Determine whether the sequence converges or diverges. If it converges, find the limit.¹

$$a_n = \frac{4^n}{1 + 9^n}$$

Let's just compute the limit as $n \rightarrow \infty$. If the limit exists, the sequence converges; if the limit does not exist, the sequence diverges.

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{4^n}{1 + 9^n} &= \lim_{n \rightarrow \infty} \frac{\frac{4^n}{9^n}}{\frac{1}{9^n} + \frac{9^n}{9^n}} \\ &= \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{9}\right)^n}{\left(\frac{1}{9}\right)^n + 1} \end{aligned}$$

Since $\lim_{n \rightarrow \infty} r^n = 0$ if $-1 < r < 1$, we get

$$\begin{aligned} &= \frac{0}{0 + 1} \\ &= 0 \end{aligned}$$

So the sequence $a_n = \frac{4^n}{1 + 9^n}$ is convergent.

¹Stewart, *Calculus, Early Transcendentals*, p. 704, #30.