Determine whether the sequence converges or diverges. If it converges, find the limit.[1]

$$a_n = \frac{(-1)^{n+1}\, n}{n + \sqrt{n}}$$

Let's compute the limit of the sequence for odd values of n (so $(-1)^{n+1} = 1$) and for even values of n (so $(-1)^{n+1} = -1$). If both limits exist and are equal, then the sequence is convergent. Otherwise, the sequence is divergent.

If n is odd, $a_n = \frac{n}{n+\sqrt{n}}$ and we have

$$\begin{aligned}
&\lim_{n\to\infty} \frac{n}{n + \sqrt{n}} \\
&= \lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{n}{n} + \frac{\sqrt{n}}{n}} \\
&= \lim_{n\to\infty} \frac{1}{1 + \frac{1}{\sqrt{n}}} \\
&= \frac{1}{1+0} \\
&= 1
\end{aligned}$$

now if n is even, $a_n = -\frac{n}{n+\sqrt{n}}$ and we have

$$\begin{aligned}
&\lim_{n\to\infty} -\frac{n}{n + \sqrt{n}} \\
&= -\lim_{n\to\infty} \frac{n}{n + \sqrt{n}} \\
&= -1
\end{aligned}$$

So the sequence $a_n = \dfrac{(-1)^{n+1}\, n}{n + \sqrt{n}}$ is divergent.

[1] Stewart, *Calculus, Early Transcendentals*, p. 704, #36.