Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?[1]

$$a_n = \frac{1-n}{2+n}$$

The corresponding function for the sequence is

$$f(x) = \frac{1-x}{2+x}$$

We compute the derivative

$$\begin{aligned} f'(x) &= \frac{(2+x)\cdot -1 - (1-x)\cdot 1}{(2+x)^2} \\ &= \frac{-2-x-1+x}{(2+x)^2} \\ &= \frac{-3}{(2+x)^2} \end{aligned}$$

The derivative of the function is negative for all values of x, so the function is decreasing for all values of x and the corresponding sequence is decreasing for all values of n. Thus the sequence is decreasing and monotonic.

Now

$$\begin{aligned} \lim_{n\to\infty} \frac{1-n}{2+n} &= \lim_{n\to\infty} \frac{\frac{1}{n}-\frac{n}{n}}{\frac{2}{n}+\frac{n}{n}} \\ &= \lim_{n\to\infty} \frac{\frac{1}{n}-1}{\frac{2}{n}+1} \\ &= \frac{0-1}{0+1} \\ &= -1 \end{aligned}$$

So the sequence is bounded below by -1 and since $a_1 = 0$, the sequence is bounded above.

[1] Stewart, *Calculus, Early Transcendentals*, p. 705, #74.