Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$ *whose partial sums are given.*[1]

$$s_n = \frac{n^2 - 1}{4n^2 + 1}$$

The sum of an infinite series is the limit of the sequence of its partial sums. So

$$\begin{aligned}
\sum_{n=1}^{\infty} a_n &= \lim_{n\to\infty} s_n \\
&= \lim_{n\to\infty} \frac{n^2 - 1}{4n^2 + 1} \\
&= \lim_{n\to\infty} \frac{\frac{n^2}{n^2} - \frac{1}{n^2}}{\frac{4n^2}{n^2} + \frac{1}{n^2}} \\
&= \lim_{n\to\infty} \frac{1 - \frac{1}{n^2}}{4 + \frac{1}{n^2}} \\
&= \frac{1}{4}
\end{aligned}$$

Thus, $\displaystyle\sum_{n=1}^{\infty} a_n = \frac{1}{4}$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 715, #4.