

Determine whether the series is convergent or divergent. If it is convergent, find its sum.¹

$$\sum_{k=0}^{\infty} (\sqrt{2})^{-k}$$

Note that

$$\begin{aligned}\sum_{k=0}^{\infty} (\sqrt{2})^{-k} &= \sum_{k=0}^{\infty} \frac{1}{(\sqrt{2})^k} \\ &= \sum_{k=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^k\end{aligned}$$

So this series is geometric with $r = \frac{1}{\sqrt{2}}$ and $a_1 = 1$. Thus the series is convergent and

$$\begin{aligned}\sum_{k=0}^{\infty} (\sqrt{2})^{-k} &= \frac{1}{1 - \frac{1}{\sqrt{2}}} \\ &= \frac{\sqrt{2}}{\sqrt{2} - 1}\end{aligned}$$

¹Stewart, *Calculus, Early Transcendentals*, p. 716, #38.