Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.[1]

$$\sum_{n=4}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$$

Let's expand the nth partial sum of the series

$$\begin{aligned}
s_n &= \left(\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{4+1}}\right)+\left(\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{5+1}}\right)+\left(\frac{1}{\sqrt{6}}-\frac{1}{\sqrt{6+1}}\right)+\dots \\
&\qquad \dots+\left(\frac{1}{\sqrt{n-1}}-\frac{1}{\sqrt{n-1+1}}\right)+\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right) \\
&= \left(\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{5}}\right)+\left(\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{6}}\right)+\left(\frac{1}{\sqrt{6}}-\frac{1}{\sqrt{7}}\right)+\dots+\left(\frac{1}{\sqrt{n-1}}-\frac{1}{\sqrt{n}}\right)+\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right) \\
&= \frac{1}{\sqrt{4}}-\frac{1}{\sqrt{n+1}}
\end{aligned}$$

We have

$$\begin{aligned}
\sum_{n=4}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right) &= \lim_{n\to\infty}\left(\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{n+1}}\right) \\
&= \frac{1}{2}
\end{aligned}$$

Thus, the limit of the sequence of partial sums exists, so the series is convergent and in fact,

$$\sum_{n=4}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)=\frac{1}{2}$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 716, #46.