*Find the values of $x$ for which the series converges. Find the sum of the series for those values of $x$*[1]

$$\sum_{n=1}^{\infty} (x+2)^n$$

Note that

$$\sum_{n=1}^{\infty} (x+2)^n = (x+2)^1 + (x+2)^2 + (x+2)^3 + (x+2)^4 + \dots$$

so the series is geometric with $r = (x+2)$ and $a_1 = (x+2)$.

The series is convergent if $|r| < 1$ or

$$\begin{aligned} |x+2| &< 1 \\ -1 < x+2 &< 1 \\ -3 < x &< -1 \end{aligned}$$

Thus, the series is convergent if $-3 < x < -1$ and

$$\begin{aligned} \sum_{n=1}^{\infty} (x+2)^n &= \frac{x+2}{1-(x+2)} \\ &= \frac{x+2}{-x-1} \end{aligned}$$

[1]Stewart, *Calculus, Early Transcendentals*, p. 716, #58.