

Calculus II, Section 11.3, #14
The Integral Test and Estimates of Sums

Determine whether the series is convergent or divergent.¹

$$1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \dots$$

The series can be written

$$\frac{1}{1\sqrt{1}} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \dots = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

So our series is a p -series with $p = \frac{3}{2} > 1$, and is therefore convergent.

¹Stewart, *Calculus, Early Transcendentals*, p. 726, #14.