Determine whether the series converges or diverges.[1]

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$$

We suspect convergence because the terms of the series are similar to $\sum \frac{n}{n^3} = \sum \frac{1}{n^2}$, which is a convergent power series with $p = 2$.

Because we suspect convergence like $\sum \frac{1}{n^2}$, let's try to show that $\frac{n-1}{n^3+1} < \frac{1}{n^2}$ so we can apply the Comparison Test.

First, note that both $a_n = \frac{n-1}{n^3+1}$ and $b_n = \frac{1}{n^2}$ are series with positive terms (at least after the first term for a_n.

We know

$$n - 1 < n \qquad \text{(This is true because we know } n \text{ is a positive number.)}$$

so

$$\frac{n-1}{n^3+1} < \frac{n}{n^3+1} \qquad \text{(This is true because we know } n^3+1 \text{ is a positive number.)}$$

but

$$\frac{n}{n^3+1} < \frac{n}{n^3}$$

(This is true because the numerators are the same, but the denominator of $\frac{n}{n^3}$ is smaller than the denominator of $\frac{n}{n^3+1}$.)

and

$$\frac{n}{n^3} \le \frac{1}{n^2}$$

(We know they are equal, but we write $\le$ to keep the string of inequalities.)

We have shown

$$\frac{n-1}{n^3+1} \le \frac{1}{n^2}$$

so by comparison with known convergent p-series $\sum \frac{1}{n^2}$, our series $\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$ is convergent.

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[1]Stewart, *Calculus, Early Transcendentals*, p. 731, #6.