Use the sum of the first 10 terms to approximate the sum of the series. Estimate the error.[1]

$$\sum_{n=1}^{\infty} \frac{\mathrm{e}^{1/n}}{n^4}$$

The tenth partial sum of the series is

$$\sum_{n=1}^{10} \frac{\mathrm{e}^{1/n}}{n^4} = \frac{\mathrm{e}^{1/1}}{1^4} + \frac{\mathrm{e}^{1/2}}{2^4}\frac{\mathrm{e}^{1/3}}{3^4} + \cdots + \frac{\mathrm{e}^{1/10}}{10^4} \approx 2.847476$$

$$\sum_{n=1}^{10} \frac{e^{1/n}}{n^4} \approx 2.8474761472372812231159119038501536711113$$

Computed by Wolfram|Alpha

To estimate the error, note that the corresponding function $f(x) = \frac{\mathrm{e}^{1/x}}{x^4}$ is continuous, positive, and decreasing on the interval $[1,\infty)$. From the Remainder Estimate for the Integral Test, we get

$$\begin{aligned} R_{10} &\le \int_{10}^{\infty} \frac{\mathrm{e}^{1/x}}{x^4}\,\mathrm{d}x \\ &\approx 0.000359364 \end{aligned}$$

Thus, the error in the estimation of the sum is less than 0.0003593.

To get the approximation, we used the TI-83 with `fnInt(e^(1/x)/(x^4),x,10,100000)`. WolframAlpha gave

$$\int_{10}^{\infty} \frac{e^{1/x}}{x^4}\,dx = \frac{181\sqrt[10]{e}}{100} - 2 \approx 0.00035936$$

Computed by Wolfram|Alpha

[1]Stewart, *Calculus, Early Transcendentals*, p. 731, #34.