Test the series for convergence or divergence.[1]

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$$

We have

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}} = -\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \cdots$$

Since some of the terms of this series are negative, none of the Integral Test, Comparison Test, or Limit Comparison Test are applicable. The terms do alternate between positive and negative, so let's try the Alternating Series Test.

For our series, we have $a_n = \frac{(-1)^{n+1}}{\sqrt{n+1}} = (-1)^{n+1} \frac{1}{\sqrt{n+1}}$. So $b_n = \frac{1}{\sqrt{n+1}}$, and $b_n > 0$ for $n \geq 0$.

To show that b_n is decreasing, *i.e.*, $b_{n+1} \leq b_n$, consider

$$\begin{aligned}
n+1 &\geq n \qquad \text{(Since } n \geq 0.) \\
n+1+1 &\geq n+1 \\
\frac{1}{n+2} &\leq \frac{1}{n+1} \qquad \text{(Since the reciprocal function } \tfrac{1}{x} \text{ is decreasing, the sense of the inequality switches when we apply it to both sides.)} \\
\sqrt{\frac{1}{n+2}} &\leq \sqrt{\frac{1}{n+1}} \qquad \text{(Since the square root function } \sqrt{x} \text{ is increasing, the sense of the inequality stays the same when we apply it to both sides.)} \\
\frac{1}{\sqrt{n+2}} &\leq \frac{1}{\sqrt{n+1}}
\end{aligned}$$

So the b_n are decreasing. (Alternatively, we can work with the corresponding function $f(x) = \frac{1}{\sqrt{n+1}}$ and show that the derivative $f'(x) = -\frac{1}{2(x+1)^{3/2}} < 0$.)

Finally, we compute

$$\begin{aligned}
\lim_{n\to\infty} b_n &= \lim_{n\to\infty} \frac{1}{\sqrt{n+1}} \\
&= \lim_{n\to\infty} \frac{\frac{1}{n^2}}{\frac{\sqrt{n+1}}{n^2}} \\
&= \lim_{n\to\infty} \frac{\frac{1}{n^2}}{\sqrt{\frac{n+1}{n}}} \\
&= \lim_{n\to\infty} \frac{\frac{1}{n^2}}{\sqrt{1+\frac{1}{n}}} \\
&= \frac{0}{\sqrt{1+0}} \\
&= 0
\end{aligned}$$

So the b_n are positive, decreasing, and have a limit of zero as n heads towards infinity. Thus, our series

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$$

is convergent by the Alternating Series Test.

[1] Stewart, *Calculus, Early Transcendentals*, p. 736, #6.