Approximate the sum of the series correct to four decimal places.[1]

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \cdot 4^n}$$

Our series is clearly alternating, with $b_n = \frac{1}{n \cdot 4^n}$ both positive and decreasing. Also, $\lim_{n\to\infty} \frac{1}{n \cdot 4^n} = 0$, so our series is a convergent alternating series, and the Alternating Series Estimation Theorem applies.

Note that

$$b_5 = \frac{1}{5 \cdot 4^5} \approx 0.0001953$$

which is larger than the desired 0.0001.

$$b_6 = \frac{1}{6 \cdot 4^6} \approx 0.0000407$$

which is smaller than the desired 0.0001, so we compute

$$\begin{aligned}\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n \cdot 4^n} \approx s_5 &= \sum_{n=1}^{5} (-1)^{n-1} \frac{1}{n \cdot 4^n} \\ &= \frac{1}{1 \cdot 4^1} - \frac{1}{2 \cdot 4^2} + \frac{1}{3 \cdot 4^3} - \frac{1}{4 \cdot 4^4} + \frac{1}{5 \cdot 4^5} \\ &\approx 0.223177\end{aligned}$$

Adding b_6 to this approximation *does* change the 4th decimal place, so we compute another term

$$b_7 = \frac{1}{7 \cdot 4^7} \approx 0.00000872$$

$$\begin{aligned}\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n \cdot 4^n} \approx s_6 &= \sum_{n=1}^{6} (-1)^{n-1} \frac{1}{n \cdot 4^n} \\ &= \frac{1}{1 \cdot 4^1} - \frac{1}{2 \cdot 4^2} + \frac{1}{3 \cdot 4^3} - \frac{1}{4 \cdot 4^4} + \frac{1}{5 \cdot 4^5} - \frac{1}{6 \cdot 4^6} \\ &\approx 0.223136\end{aligned}$$

Adding b_7 to this approximation does not change the 4th decimal place, so this approximation of the sum of the series is correct to four decimal places, *i.e.*,

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n} \approx 0.2231$$

[1] Stewart, *Calculus, Early Transcendentals*, p. 736, #30.