Use the Ratio Test to determine whether the series is convergent or divergent.[1]

$$\frac{2}{3}+\frac{2\cdot 5}{3\cdot 5}+\frac{2\cdot 5\cdot 8}{3\cdot 5\cdot 7}+\frac{2\cdot 5\cdot 8\cdot 11}{3\cdot 5\cdot 7\cdot 9}+\cdots$$

We can rewrite our series as

$$\frac{2}{3}+\frac{2\cdot 5}{3\cdot 5}+\frac{2\cdot 5\cdot 8}{3\cdot 5\cdot 7}+\frac{2\cdot 5\cdot 8\cdot 11}{3\cdot 5\cdot 7\cdot 9}+\cdots=\sum_{n=1}^{\infty}\frac{2\cdot 5\cdot 8\cdot 11\cdot\cdots\cdot(3n-1)}{3\cdot 5\cdot 7\cdot 9\cdot\cdots\cdot(2n+1)}$$

and compute the limit

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\frac{\frac{2\cdot 5\cdot 8\cdot 11\cdot\cdots\cdot(3(n+1)-1)}{3\cdot 5\cdot 7\cdot 9\cdot\cdots\cdot(2(n+1)+1)}}{\frac{2\cdot 5\cdot 8\cdot 11\cdot\cdots\cdot(3n-1)}{3\cdot 5\cdot 7\cdot 9\cdot\cdots\cdot(2n+1)}}\right| \\
&= \lim_{n\to\infty}\left|\frac{2\cdot 5\cdot 8\cdot 11\cdot\cdots\cdot(3n-1)\,(3n+2)}{3\cdot 5\cdot 7\cdot 9\cdot\cdots\cdot(2n+1)\,(2n+3)}\cdot\frac{3\cdot 5\cdot 7\cdot 9\cdot\cdots\cdot(2n+1)}{2\cdot 5\cdot 8\cdot 11\cdot\cdots\cdot(3n-1)}\right| \\
&= \lim_{n\to\infty}\left|\frac{3n+2}{2n+3}\right| \\
&= \frac{3}{2}
\end{aligned}$$

Since the limit exists and is greater than one, the series

$$\sum_{n=1}^{\infty}\frac{2\cdot 5\cdot 8\cdot 11\cdot\cdots\cdot(3n-1)}{3\cdot 5\cdot 7\cdot 9\cdot\cdots\cdot(2n+1)}$$

is divergent by the Ratio Test.

[1] Stewart, *Calculus, Early Transcendentals*, p. 743, #22.