Use the Root Test to determine whether the series is convergent or divergent.[1]

$$\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1} \right)^{5n}$$

We compute the limit

$$\begin{aligned}
\lim_{n\to\infty} \sqrt[n]{|a_n|} &= \lim_{n\to\infty} \sqrt[n]{\left| \left(\frac{-2n}{n+1} \right)^{5n} \right|} \\
&= \lim_{n\to\infty} \left| \left(\frac{-1 \cdot 2 \cdot n}{n+1} \right)^{5} \right| \\
&= \lim_{n\to\infty} \left| \frac{(-1)^5 \cdot 2^5 \cdot n^5}{(n+1)^5} \right| \\
&= \lim_{n\to\infty} \frac{2^5 \cdot n^5}{(n+1)^5} \\
&= \lim_{n\to\infty} \frac{\frac{2^5 \cdot n^5}{n^5}}{\frac{(n+1)^5}{n^5}} \\
&= \lim_{n\to\infty} \frac{32}{\left(\frac{n+1}{n}\right)^5} \\
&= \lim_{n\to\infty} \frac{32}{\left(1+\frac{1}{n}\right)^5} \\
&= \frac{32}{(1+0)^5} \\
&= 32
\end{aligned}$$

Since the limit exists and is greater than one, the series

$$\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1} \right)^{5n}$$

is divergent by the Root Test.

[1] Stewart, *Calculus, Early Transcendentals*, p. 743, #28.