The function A defined by[1]

$$A(x) = 1 + \frac{x^3}{2\cdot 3} + \frac{x^6}{2\cdot 3\cdot 5\cdot 6} + \frac{x^9}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9} + \cdots$$

is called an Airy function *after the English mathematician and astronomer Sir George Airy (1801-1892).*

(a) *Find the domain of the Airy function.*

If we can write the function as a power series, then the domain of the function is the interval of convergence. Note that

$$A(x) = 1 + \frac{x^3}{2\cdot 3} + \frac{x^6}{2\cdot 3\cdot 5\cdot 6} + \frac{x^9}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9} + \cdots$$

Since the first term doesn't *seem* to fit the pattern of the other terms, we'll treat this as

$$= 1 + \sum_{n=1}^{\infty} \frac{x^{3n}}{2\cdot 3\cdot 5\cdot 6\cdot \cdots \cdot (3n-1)(3n)}$$

We apply the Ratio Test to the sum

$$\begin{aligned}
&\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \\
&= \lim_{n\to\infty} \left|a_{n+1}\cdot\frac{1}{a_n}\right| \\
&= \lim_{n\to\infty} \left|\frac{x^{3(n+1)}}{2\cdot 3\cdot 5\cdot 6\cdot \cdots \cdot (3(n+1)-1)(3(n+1))}\cdot\frac{2\cdot 3\cdot 5\cdot 6\cdot \cdots \cdot (3n-1)(3n)}{x^{3n}}\right| \\
&= \lim_{n\to\infty} \left|\frac{x^{3n+3}}{x^{3n}}\cdot\frac{2\cdot 3\cdot 5\cdot 6\cdot \cdots \cdot (3n-1)(3n)}{2\cdot 3\cdot 5\cdot 6\cdot \cdots \cdot (3n+2)(3n+3)}\right| \\
&= \lim_{n\to\infty} \left|x^3\cdot\frac{2\cdot 3\cdot 5\cdot 6\cdot \cdots \cdot (3n-1)(3n)}{2\cdot 3\cdot 5\cdot 6\cdot \cdots \cdot (3n-1)(3n)(3n+2)(3n+3)}\right| \\
&= \left|x^3\right|\cdot\lim_{n\to\infty}\left|\frac{1}{(3n)(3n+2)(3n+3)}\right| \\
&= \left|x^3\right|\cdot 0 \\
&= 0
\end{aligned}$$

Thus the series is convergent for all x and the domain is $(-\infty,\infty)$. (Note that the "1+" does not affect the convergence nor the domain because it does not involve x in any way.)

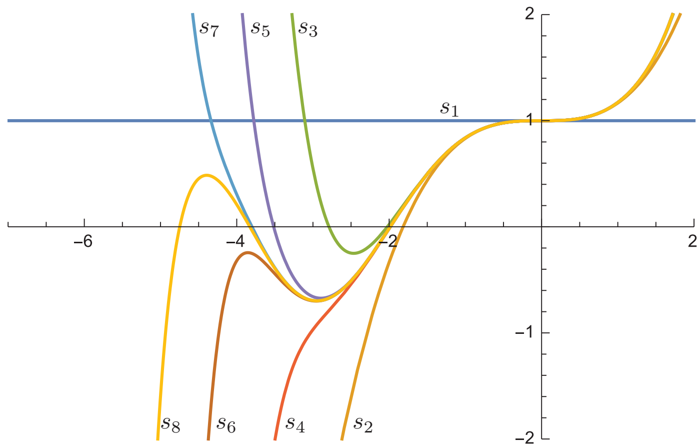
(b) *Graph the first several partial sums on a common screen.*

The partial sums are

$$\begin{aligned}
s_1 &= 1 \\
s_2 &= 1 + \frac{x^3}{2\cdot 3} \\
s_3 &= 1 + \frac{x^3}{2\cdot 3} + \frac{x^6}{2\cdot 3\cdot 5\cdot 6} \\
s_4 &= 1 + \frac{x^3}{2\cdot 3} + \frac{x^6}{2\cdot 3\cdot 5\cdot 6} + \frac{x^9}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9} \\
s_5 &= 1 + \frac{x^3}{2\cdot 3} + \frac{x^6}{2\cdot 3\cdot 5\cdot 6} + \frac{x^9}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9} + \frac{x^{12}}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9\cdot 11\cdot 12}
\end{aligned}$$

and so on.

[1] Stewart, *Calculus, Early Transcendentals*, p. 752, #36.



(c) *If your CAS has built-in Airy functions, graph A on the same screen as the partial sums in part (b) and observe how the partial sums approximate A.*

After lengthy experiments, we were not able to get WolframAlpha to generate a graph of the function. The program has a couple of built-in functions that are listed as Airy functions, but neither of them matched our work from parts (a) or (b).