Find a power series representation for the function and determine the interval of convergence.[1]

$$f(x) = \frac{5}{1-4x^2}$$

The form for the sum of a geometric series with first term 1 and common ratio x is

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

where the series will converge for $|x| < 1$, so we will try to write our function in this form.

$$\begin{aligned} f(x) &= \frac{5}{1-4x^2} \\ &= 5 \cdot \frac{1}{1-(4x^2)} \\ &= 5 \cdot \sum_{n=0}^{\infty} \left(4x^2\right)^n \end{aligned}$$

so the series has first term 1 and common ratio $4x^2$. Simplifying

$$= 5 \cdot \sum_{n=0}^{\infty} 4^n x^{2n}$$

The series converges for

$$\begin{aligned} \left|4x^2\right| &< 1 \\ 4\left|x^2\right| &< 1 \\ \left|x^2\right| &< \frac{1}{4} \\ (|x|)^2 &< \frac{1}{4} \\ |x| &< \frac{1}{2} \\ -\frac{1}{2} < x &< \frac{1}{2} \end{aligned}$$

Thus the radius of convergence is $R = \frac{1}{2}$, and because we obtained the interval of convergence from a geometric form, we have $I = \left(-\frac{1}{2}, \frac{1}{2}\right)$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 757, #4.