Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x)$ centered at the given value of a.[1]

$$f(x) = \cos^2(x), \quad a = 0$$

The general form for a Taylor series is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

It is often helpful to organize our work in a table. For this problem, we can stop finding entries in the table when we get four nonzero values for $f^{(n)}(0)$.

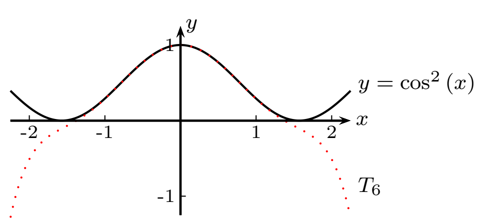
n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$\cos^2(x)$	1
1	$-2\cos(x)\sin(x) = -\sin(2x)$	0
2	$-2\cos(2x)$	-2
3	$4\sin(2x)$	0
4	$8\cos(2x)$	8
5	$-16\sin(2x)$	0
6	$-32\cos(2x)$	-32

Thus,

$$\begin{aligned} f(x) &\approx \frac{1}{0!}x^0 - \frac{2}{2!}x^2 + \frac{8}{4!}x^4 - \frac{32}{6!}x^6 \\ &= 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 \end{aligned}$$

We've found T_6, the 6th degree Taylor polynomial of $f(x) = \cos^2(x)$ at 0.

Here we've graphed the function $f(x) = \cos^2(x)$ in black and T_6 in dotted red.



From the graph, it seems that T_6 is a good approximation to $y = \cos^2(x)$ between $x = -1$ and $x = 1$; more terms of the Taylor polynomial will extend this interval to the left and the right.

[1] Stewart, *Calculus, Early Transcendentals*, p. 771, #10.