

Calculus II, Section 11.11, #8
Applications of Taylor Polynomials

Find the Taylor polynomial $T_3(x)$ for the function f centered at the number a . Graph f and T_3 on the same screen.¹

$$f(x) = x \cos(x), \quad a = 0$$

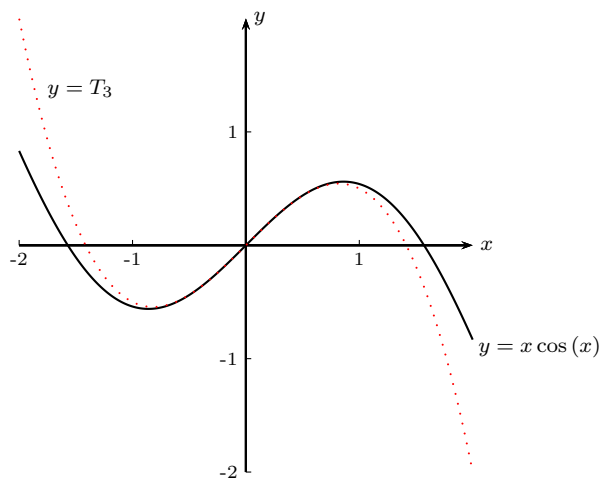
Let's make a table of the values of f , f' , f'' , etc.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$x \cos(x)$	0
1	$\cos(x) - x \sin(x)$	1
2	$-x \cos(x) - 2 \sin(x)$	0
3	$x \sin(x) - 3 \cos(x)$	-3
4	$x \cos(x) + 4 \sin(x)$	0
5	$-x \sin(x) + 5 \cos(x)$	5

So

$$\begin{aligned} T_3 &= \sum_{n=0}^3 \frac{f^{(n)}(0)}{n!} x^n \\ &= 0 + \frac{1}{1!}x + 0 + \frac{-3}{3!}x^3 \\ &= x - \frac{1}{2}x^3 \end{aligned}$$

We'll graph f in solid black, and T_3 in dotted red.



¹Stewart, *Calculus, Early Transcendentals*, p. 780, #8.