(a) *Approximate f by a Taylor polynomial with degree n at the number a.*[1]

$$f(x) = \ln(1+2x), \quad a = 1, \quad n = 3, \quad 0.5 \le x \le 1.5$$

Let's make a table of the values of f, f', f'', etc.

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	$\ln(1+2x)$	$\ln(3)$
1	$\frac{2}{1+2x}$	$\frac{2}{3}$
2	$-\frac{4}{(1+2x)^2}$	$-\frac{4}{9}$
3	$\frac{16}{(1+2x)^3}$	$\frac{16}{27}$
4	$\frac{-96}{(1+2x)^4}$	$-\frac{32}{27}$

So

$$\begin{aligned} T_3 &= \sum_{n=0}^{3} \frac{f^{(n)}(0)}{n!}(x-1)^n \\ &= \frac{\ln(3)}{0!}(x-1)^0 + \frac{2/3}{1!}(x-1)^1 + \frac{-4/9}{2!}(x-1)^2 + \frac{16/27}{3!}(x-1)^3 \\ &= \ln(3) + \frac{2}{3}(x-1) - \frac{2}{9}(x-1)^2 + \frac{8}{81}(x-1)^3 \end{aligned}$$

(b) *Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_3(x)$ when x lies in the given interval.*

Taylor's Inequality gives us

$$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1} \text{ for } |x-a| \le d$$

where $\left|f^{(n+1)}(x)\right| \le M$ for $|x-a| \le d$.

To find M, let's use our calculator to graph $y = \left|f^{(4)}(x)\right|$ on the interval $0.5 \le x \le 1.5$ and find the maximum value. From the graphing calculator, we get $M = 6$; see Figure 1. Now we'll graph $y = \frac{6}{4!}|x-1|^4$ and find its maximum value. From the graphing calculator, the maximum value is 0.015625; see Figure 2.

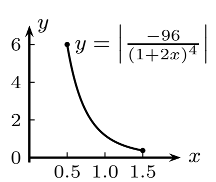

Figure 1: Finding M.

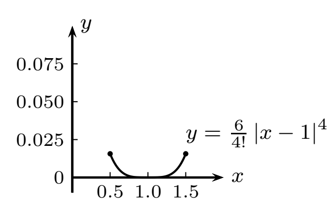

Figure 2: Finding $|R_3(x)|$.

Thus, whenever $0.5 \le x \le 1.5$, the approximation given by $T_3(x)$ is within 0.015625 of the actual value of $f(x) = \ln(1+2x)$.

[1]Stewart, *Calculus, Early Transcendentals*, p. 781, #18.

(c) *Check your result in part (b) by graphing* $|R_3(x)|$.

See Figure 2.

Here is a graph showing $f(x) = \ln(1+2x)$ in solid black and $T_3(x) = \ln(3) + \frac{2}{3}(x-1) - \frac{2}{9}(x-1)^2 + \frac{8}{81}(x-1)^3$ in dotted red.