Use the Alternating Series Estimation Theorem or Taylor's Inequality to estimate the range of values of x for which the given approximation is accurate to within the stated error. Check your answer graphically.[1]

$$\cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}, \quad |\text{error}| < 0.005$$

We know

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)^n}$$

so we will use the Alternating Series Estimation Theorem.

The Alternating Series Estimation Theorem says that the error is smaller than the first excluded term, so for our estimate of $\cos(x)$, we have

$$\begin{aligned} \left|-\frac{x^6}{6!}\right| &< 0.005 \\ \left|\frac{x^6}{720}\right| &< 0.005 \\ \left|x^6\right| &< 3.6 \\ |x| &< (3.6)^{1/6} \\ -1.2380 < x &< 1.2380 \end{aligned}$$

When we use the estimate $\cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$ we are guaranteed to be within 0.005 of the correct value of $\cos(x)$ whenever $-1.2380 < x < 1.2380$.

[1] Stewart, *Calculus, Early Transcendentals*, p. 781, #28.

This graph shows our approximation $y = 1 - \frac{x^2}{2} + \frac{x^4}{24}$ in black along with $y = \cos(x) + 0.005$ and $y = \cos(x) - 0.005$ in red. We can see that the approximation is within ± 0.005 whenever $-1.2380 < x < 1.2380$.