A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.¹

(a) At what rate is his distance from second base decreasing when he is halfway to first base?

If we let \( x = \) distance batter has run at time \( t \) and \( D = \) distance from second base to the batter at time \( t \), then we know \( \frac{dx}{dt} = 24 \) and we want \( \frac{dD}{dt} \) when \( x = 45 \).

From Pythagorean Theorem
\[
D^2 = (90 - x)^2 + 90^2
\]
differentiating with respect to \( t \)
\[
2D \frac{dD}{dt} = 2 (90 - x)(-1) \frac{dx}{dt} + 0
\]
\[
D \frac{dD}{dt} = -(90 - x) \frac{dx}{dt}
\]
Now, when \( x = 45 \),
\[
D = \sqrt{(90 - 45)^2 + 90^2}
\]
\[
= \sqrt{(45)^2 + 90^2}
\]
and substituting
\[
\frac{\sqrt{(45)^2 + 90^2}}{dt} = -(90 - 45) \cdot 24
\]
\[
\frac{dD}{dt} = \frac{-45 \cdot 24}{\sqrt{(45)^2 + 90^2}}
\]
\[
\approx -10.73 \text{ ft/s}
\]

Thus, when the batter is halfway to first base, the distance between second base and the batter is decreasing at the rate of about 10.73 ft/s.

¹Stewart, Calculus, Early Transcendentals, p. 249, #20.
(b) At what rate is his distance from third base increasing at the same moment?

If we let \( x \) = distance batter has run at time \( t \) and \( D = \text{distance from third base to the batter at time } t \), then we know \( \frac{dx}{dt} = 24 \) and we want \( \frac{dD}{dt} \) when \( x = 45 \).

From Pythagorean Theorem
\[
D^2 = x^2 + 90^2
\]
differentiating with respect to \( t \)
\[
2D \cdot \frac{dD}{dt} = 2x \frac{dx}{dt} + 0
\]
\[
\frac{dD}{dt} = \frac{x}{D} \frac{dx}{dt}
\]

Now, when \( x = 45 \),
\[
D = \sqrt{45^2 + 90^2}
\]
and substituting
\[
\frac{dD}{dt} = \frac{45 \cdot 24}{\sqrt{45^2 + 90^2}}
\]
\[
\approx 10.73 \text{ ft/s}
\]

Thus, when the batter is halfway to first base, the distance between third base and the batter is increasing at the rate of about 10.73 ft/s.