1. Find the sum of the solutions to the equations $x^2 - 5x - 6 = 0$ and $x^2 + 4x + 3 = 0$ which DO NOT satisfy both equations at once.

A. -2 B. -1 C. 1 D. 2 E. 3

2. Four consecutive integers are substituted in every possible order for a, b, c, and d. Find the difference between the maximum and minimum values of ab + cd.

A. 1 B. 2 C. 3 D. 4 E. 5

3. The product of a number and b more than its reciprocal is y ($b > 0$). Express the number in terms of b and y.

A. $\frac{y-1}{b}$ B. $\frac{y+1}{b}$ C. $\frac{y}{b}-1$ D. $\frac{y}{b}+1$ E. $1-\frac{y}{b}$

4. If $f(x) = x^2 - x + 2$, find the sum of all x values satisfying $f(x-2) = 22$.

A. -5 B. -1 C. 1 D. 3 E. 5

5. Sue bikes 2.5 times as fast as Joe runs, and in 1 hr they cover a total of 42 miles. What is their combined distance if Sue bikes for 0.5 hr and Joe runs for 1.5 hr?

A. 27 B. 30 C. 33 D. 36 E. 39

6. The equation $a^4 + b^3 + c^2 = 2009$ (a, b, c positive integers) has a solution in which a and b are both perfect squares. Find $a + b + c$.

A. 20 B. 21 C. 32 D. 45 E. 50

7. How many 3-digit numbers have one digit equal to the average of the other 2?

A. 96 B. 100 C. 112 D. 120 E. 121

8. A rectangular solid has integer dimensions with length ≥ width ≥ height and volume 60. How many such distinct solids are there?

A. 6 B. 8 C. 10 D. 12 E. 15

9. $\frac{2 \sin x}{\cos x - \sin x \tan x} =$ A. tan 2x B. cot 2x C. tan x D. cot x E. sec x

10. If $x + \frac{1}{y} = 12$ and $y + \frac{1}{x} = \frac{3}{8}$, find the largest value of xy.

A. 1/4 B. 1/2 C. 1 D. 2 E. 4

11. In quadrilateral ABCD, P is a point in its interior such that ∠DAP = ∠BAP, ∠CBP = ∠ABP, and ∠APB = 90°. Quadrilateral ABCD must be which of the following?

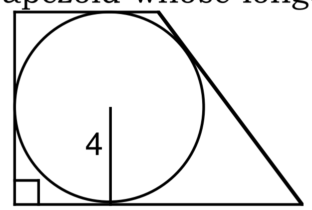
A. trapezoid B. parallelogram C. rectangle D. B and C E. none of these

12. The sum of the squares of the three roots of $P(x) = 2x^3 - 6x^2 + 3x + 5$ is

A. 3 B. 6 C. 30 D. 33 E. 39

13. The value of $4^{\log_2\left(2^{1/4}2^{1/8}2^{1/16}\cdots\right)}$ is A. 1 B. $\sqrt{2}$ C. 2 D. $2\sqrt{2}$ E. 4

14. The figure shows a circle of radius 4 inscribed in a trapezoid whose longer base is three times the radius of the circle. Find the area of the trapezoid.



A. 72 B. 74 C. 76 D. 78 E. 80

15. In how many ways can six computers be networked so that each computer is directly connected to exactly two other computers, and all computers are connected directly or indirectly?

A. 24 B. 36 C. 48 D. 60 E. 120

16. The integer r > 1 is both the common ratio of an integer geometric sequence and the common difference of an integer arithmetic sequence. Summing corresponding terms of the sequences yields 7, 26, 90, …. The value of r is

A. 2 B. 4 C. 6 D. 8 E. 12

17. A hallway has 8 offices on one side and 5 offices on the other side. A worker randomly starts in one office and randomly goes to a second and then a third office (all three different). Find the probability that the worker crosses the hallway at least once.

A. $\frac{7}{13}$ B. $\frac{8}{13}$ C. $\frac{9}{13}$ D. $\frac{10}{13}$ E. $\frac{11}{13}$

18. Let S = {123, 124, …, 987} be the set of all three-digit numbers with distinct nonzero digits. For which number N below does S contain at least two different numbers with the same three digits, both divisible by N?

A. 31 B. 37 C. 39 D. 41 E. 43

19. In square ABCD, AB = 10. The square is rotated 45° around point P, the intersection of $\overline{AC}$ and $\overline{BD}$. Find the area of the union of ABCD and the rotated square to the nearest square unit.

A. 117 B. 119 C. 121 D. 123 E. 125

20. The sum of the 100 consecutive perfect squares starting with a^2 ($a > 0$) equals the sum of the next 99 consecutive perfect squares. Find a. Write your answer in the corresponding blank on the answer sheet.

1. (E)
2. (D)
3. (A)
4. (E)
5. (C)
6. (D)
7. (E)
8. (C)
9. (A)
10. (D)
11. (A) or (E)
12. (B)
13. (C)
14. (A)
15. (D)
16. (B)
17. (D)
18. (B)
19. (A)
20. 19,701

1. (E) The solutions to $x^2 - 5x - 6 = 0$, are -1 and 6; the solutions to $x^2 + 4x + 3 = 0$ are -3 and -1. -1 is a solution to both, -3 and 6 aren't.
2. (D) True for any four consecutive integers.
3. (A) $x(\frac{1}{x} + b) = y \Rightarrow x = \dfrac{y-1}{b}$
4. (E) $(x-2)^2 - (x-2) + 2 = 22 \Rightarrow x^2 - 5x - 14 = 0 \Rightarrow x = -2, 7$
5. (C) $r_s = 2.5r_j$, $r_j + r_s = 42 \Rightarrow r_j + 2.5r_j = 42 \Rightarrow r_j = 12$, $r_s = 30$; $0.5(30) + 1.5(12) = 33$
6. (D) $4^4 + 9^3 + 32^2 = 2009$
7. (E) digits (permutations): 0-2-1 (4), 0-4-2 (4), 0-6-3 (4), 0-8-4 (4), X-X-X (9), 1-3-2 (6), 1-5-3 (6), etc.
8. (C) $60 = 1 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \Rightarrow 1 \times 1 \times 60$, $1 \times 2 \times 30$, $1 \times 3 \times 20$, $1 \times 4 \times 15$, , $1 \times 5 \times 12$, $1 \times 6 \times 10$, $2 \times 2 \times 15$, $2 \times 3 \times 10$, $2 \times 5 \times 6$, $3 \times 4 \times 5$
9. (A) $\dfrac{2\sin x}{\cos x - \sin x \tan x} = \dfrac{2\sin x}{\cos x - \frac{\sin^2 x}{\cos x}} = \dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x} = \dfrac{\sin 2x}{\cos 2x} = \tan 2x$
10. (D) $\Rightarrow xy + 1 = 12y$ and $xy + 1 = \frac{3}{8}x$, subtract the equations to get $y = \frac{1}{32}x \Rightarrow x = 4, 8$ and $y = \frac{1}{8}, \frac{1}{4}$
11. (A) or (E) $\angle$DAB and $\angle$ABC are supplementary $\therefore \overline{\text{DA}} \parallel \overline{\text{BC}}$. Some textbooks define a trapezoid as a quadrilateral with *at least* on pair of parallel sides and others as *exactly* one pair of parallel sides.
12. (B)
13. (C) $2^{2\log_2(2^{1/4}2^{1/8}2^{1/16}\cdots)} = 2^{\log_2(2^{1/2}2^{1/4}2^{1/8}\cdots)} = 2^{1/2}2^{1/4}2^{1/8}\cdots = 2^{1/2+1/4+1/8+\cdots} = 2^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} = 2$
14. (A)
15. (D)
16. (B)
17. (D) $1 - P(\text{worker stays on one side}) = 1 - (\frac{8}{13} \cdot \frac{7}{12} \cdot \frac{6}{11} + \frac{5}{13} \cdot \frac{4}{12} \cdot \frac{3}{11}) = \frac{10}{13}$
18. (B)
19. (A) The additional area consists of four isosceles right triangles. The side of the original square is partitioned into the length of two legs and a hypotenuse of a 45-45-90 triangle. $10 = x + x + x\sqrt{2} \Rightarrow x = 5(2 - \sqrt{2})$. x is the length of each leg of the additional four triangles. Therefore the total area is $A = 10^2 + 4 \cdot \frac{1}{2}(5(2 - \sqrt{2}))^2 = 100 + 100(3 - 2\sqrt{2}) \approx 117$
20. 19,701