

- If  $f(x) = \cos \pi x$  and  $g(x) = 2x$ , find  $f(g(1)) - g(f(1))$ .  
A. -3                      B. -1                      C. 0                      D. 1                      E. 3
- How many different four-digit numbers can be formed by arranging the digits 2, 0, 0, and 6?  
A. 6                      B. 8                      C. 10                      D. 12                      E. 24
- If ABCD, DCEF, and FEGH are squares with A, B, C, D, E, F, G, and H all distinct points, find  $m\angle GAH + m\angle GDH + m\angle GFH$  to the nearest tenth degree.  
A.  $80^\circ$                       B.  $87.5^\circ$                       C.  $90^\circ$                       D.  $92.5^\circ$                       E.  $100^\circ$
- I sold a horse for \$200, losing 20%. I bought another horse and sold it for a 25% profit. If I broke even on the two transactions together, what was the total cost of the two horses?  
A. \$432                      B. \$450                      C. \$500                      D. \$540                      E. \$562.50
- Let  $A(m, n)$  be the set of  $n$  consecutive positive integers whose least element is  $m$ . What is the greatest integer in  $A(17, 49) \cap A(49, 17)$ ?  
A. 33                      B. 49                      C. 65                      D. 66                      E. 67
- Let  $a, b > 0$ ,  $M = \sum_{n=1}^k \ln(an) - \sum_{n=1}^k \ln(bn)$ ,  $N = e^M$ , and  $P = \sqrt[k]{N}$ . Then  $P$  equals  
A.  $\frac{a}{b}$                       B.  $a - b$                       C.  $\sqrt[k]{k(a - b)}$                       D.  $\sqrt[k]{\frac{ka}{b}}$                       E.  $e^{a/bk}$
- Which of the following imply that the real number  $x$  must be rational?  
I.  $x^5, x^7$  are both rational  
II.  $x^6, x^8$  are both rational  
III.  $x^5, x^8$  are both rational  
A. I, II only    B. I, III only    C. II, III only    D. III only    E. none of these combinations
- A positive integer less than 1000 is chosen at random. What is the probability it is a multiple of 3, but a multiple of neither 2 or 9?  
A.  $\frac{1}{10}$                       B.  $\frac{1}{9}$                       C.  $\frac{1}{8}$                       D.  $\frac{2}{9}$                       E.  $\frac{1}{3}$
- Let  $r$  and  $s$  be the solutions to the equation  $x^2 + 3x + c = 0$ . If  $r^2 + s^2 = 33$ , find the value of  $c$ .  
A. -21                      B. -12                      C. 1                      D. 12                      E. 21
- Joe must determine the greatest whole number of feet from which a ceramic ball can be dropped without breaking. He has two identical ceramic balls which he can drop from any whole numbered height he wants. If he must determine this height with no more than 12 drops, what is the greatest height for which he can determine this with certainty?  
A. 20-25 ft    B. 26-40 ft    C. 41-50 ft    D. 51-75 ft    E. more than 75 ft

11. In convex pentagon AMTYC,  $\overline{CY} \perp \overline{YT}$ ,  $\overline{MT} \perp \overline{YT}$ ,  $CY = YT = 63$ ,  $MT = 79$ ,  $AM = 39$ , and  $AC = 52$ . Find the area of the pentagon.
- A. 5487      B. 5500      C. 5525      D. 5600      E. 5624
12. The *midrange* of a set of numbers is the average of the greatest and least values in the set. For a set of six increasing nonnegative integers, the mean, the median, and the midrange are all 5. How many such sets are there?
- A. 10      B. 12      C. 20      D. 24      E. 30
13. The sum of the absolute values of all solutions of the equation  $|x^3 + 4x^2 - 6x - 22| = x^2 + 2x + 2$ , can be written in the form  $a + b\sqrt{c}$ ,  $c$  a prime. Find  $a + b + c$ .
- A. 12      B. 14      C. 16      D. 17      E. 18
14. Find the number of three-digit numbers containing no even digits which are divisible by 9.
- A. 8      B. 9      C. 10      D. 11      E. 12
15. If  $\alpha$  is the acute angle formed by the lines with equations  $y = 2x - 5$  and  $y = 1 - 3x$ , find  $\tan \alpha$ .
- A.  $\frac{1}{\sqrt{3}}$       B.  $\frac{1}{2}$       C. 1      D. 2      E.  $\sqrt{3}$
16. Find the number of points of intersection of the unit circle and the graph of the equation  $y^2 - xy - |x|y + x|x| = 0$
- A. 0      B. 1      C. 2      D. 3      E. 4
17. Suppose that for a function  $y = f(x)$ ,  $f(x) > x$  for all  $x$ . Let  $A$  be the point with  $x$ -coordinate  $a$  on the function  $y = f(x)$  and  $B$  be the point on the graph of the line  $y = x$  for which  $\overline{AB}$  is perpendicular to the line. Find an expression for the distance from  $A$  to  $B$ .
- A.  $(f(a) - a)\sqrt{2}$       B.  $a\frac{\sqrt{2}}{2}$       C.  $(f(a) - a)\frac{\sqrt{2}}{2}$       D.  $f(a) - a$       E.  $f(a)\sqrt{2}$
18. In quadrilateral PQRS,  $PQ = 1$ ,  $QR = RS = \sqrt{2}$ ,  $PS = \sqrt{3}$ , and  $QS = 2$ . If  $T$  is the point of intersection of the diagonals, find the measure in degrees of angle RTS.
- A. 45      B. 55      C. 60      D. 75      E. 105
19. Call a composite number *circumfactorable* if all of this positive integer factors greater than 1 can be placed around a circle so that any two adjacent factors have a common factor greater than 1. How many composite numbers less than 200 are not circumfactorable?
- A. 50      B. 52      C. 54      D. 56      E. 58
20. A circle contains 2006 points chosen so that the arcs between any two adjacent points are equal. Let the probability that the triangle formed is right be  $R$ , and the probability that the triangle formed is isosceles be  $I$ . Find  $|R - I|$ .
- A. 0      B.  $\frac{1}{5}$       C.  $\frac{1}{4}$       D.  $\frac{1}{3}$       E.  $\frac{1}{2}$

Test #2

AMATYC Student Mathematics League Answers

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1. E
2. A
3. C
4. B
5. C
6. A
7. B
8. B
9. B
10. E
11. A
12. A
13. E
14. D
15. C
16. D
17. C
18. D
19. D
20. A