1. A store advertises, "We pay the sales tax!" If sales tax is 8%, what discount to the buyer to the nearest tenth of a percent does this represent?

A. 7.4% B. 7.5% C. 7.6% D. 7.7% E. 7.8%

2. The lines with equations $ax + 2y = c$ and $bx - 3y = d$ are perpendicular. Find $a \cdot b$.

A. -6 B. -1.5 C. -1 D. 1.5 E. 6

3. Sue owes $12,000 on a loan. She makes monthly payments of $200, and $10 interest is added each month to her balance. In how many months is the loan paid off?

A. 60 B. 61 C. 62 D. 63 E. 64

4. The polynomial $3x^2 + 4xy - 4y^2$ can be factored as the product of two first-degree polynomials. The sum of the two factors is

A. 4x B. 4y C. 2x D. 2x + 2y E. 4x + 4y

5. The lines with equations $2x + 3y = 6$ and $x + 2y = 5$ intersect at the point (a, b). The sum a + b equals A. -2 B. -1 C. 0 D. 1 E. 2

6. A domino is a 1x2 rectangle. When 8 dominos are formed into all possible rectangles with no spaces or gaps, let P be the greatest possible perimeter and p the least possible perimeter. Find P/p.

A. 1.25 B. 1.75 C. 2 D. 2.125 E. 2.375

7. The 5-digit number 217xy has 5 different digits and a factor of 45. Find x + y.

A. 8 B. 9 C. 10 D. 11 E. 12

8. Ed and Em order sodas at the 8-12 store. After Ed drinks half of his and Em drinks 1/3 of hers, they have the same number of ounces of soda left. If the two sodas totaled 28 oz originally, how many ounces of soda total do the two of them have left?

A. 12 B. 15 C. 16 D. 18 E. 20

9. Let S = {3, 5, 7, 11, 13, 17}. How many elements of S are factors of $2^{60} - 1$?

A. 2 B. 3 C. 4 D. 5 E. 6

10. On Jan. 27, postal rates rose from 46¢ to 49¢ an ounce. Vi buys some new 49¢ stamps and some 3¢ stamps to use with her leftover 46¢ stamps. If she spends $4.10 and buys more 49¢ stamps than 3¢ stamps, how many stamps does she buy?

A. 12 B. 14 C. 16 D. 18 E. it cannot be determined

11. The equation $a^4 + b^2 + c^2 = 2014$ has a unique solution in positive integers. For this solution, find $a + b + c$. A. 56 B. 58 C. 60 D. 62 E. 64

12. Different letters are placed on the 18 faces of 3 standard 6-sided dice, one per face. Choosing 1 letter from each die, the following words can be formed: bow, boy, cot, dry, gas, hat, oat, old, one, pay, pie, red, six. Which of the following could also be spelled?

A. eat B. rap C. top D. wad E. won

13. The fraction $\frac{a}{b}$ is 0.455 when rounded to 3 decimal places. If $\frac{a+1}{b+1}$ is 0.467 when rounded to 3 decimal places, find $a + b$.

A. 63 B. 64 C. 65 D. 66 E. 67

14. If ax + b = 15 and 15x + a = b have the same unique solution, where a and b are positive integers both less than or equal to 30, find the sum of all possible values of a.

A. 28 B. 43 C. 58 D. 78 E. 93

15. If (r, s, t, u, v) satisfies the system $\begin{cases} 3r+10s+16t+30u+25v=10 \\ 4r+15s+20t+36u+36v=11 \\ 5r+20s+24t+42u+49v=20 \end{cases}$, then the value of

$6r + 25s + 28t + 48u + 64v$ is A. 33 B. 34 C. 35 D. 36 E. 37

16. In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$ and E is the point of intersection of $\overline{AC}$ and $\overline{BD}$. If the area of ΔCDE is 75 and the area of ΔABE is 48, find the area of the trapezoid.

A. 216 B. 225 C. 240 D. 243 E. 246

17. There is a unique integer N with the property that N has the 4-digit representation pqrs in base 7 and the 4-digit representation qrsp in base 9 ($p \neq 0$, $q \neq 0$). Write the base-10 representation of N in the corresponding blank on the answer sheet.

18. In *approval voting*, each voter can distribute up to 5 votes among 6 candidates. For example, you could cast 3 votes for one candidate and 2 for another, or you could cast 1 vote for each of 4 candidates (and not cast your fifth vote). In how many ways can you distribute your votes?

A. 252 B. 256 C. 462 D. 480 E. 720

19. The polynomial $P(x) = x^4 + mx^3 + nx^2 - 24x + 144$ has exactly 2 distinct integer roots, and no other roots, real or complex. Find m + n.

A. -27 B. -25 C. -23 D. -21 E. -19

20. A subset S of {1, 2, 3, …, n} is called *odd-neighbored* if for each even number k in S, if k < n then S contains both k - 1 and k + 1, and if k = n then S contains k - 1. For example, ∅, {1, 3, 5, 7}, {1, 2, 3, 5}, and {3, 4, 5, 7, 8} are all odd-neighbored subsets of {1, 2, 3, …, 8}. Find the number of nonempty odd-neighbored subsets of {1, 2, 3, …, 12}.

A. 232 B. 264 C. 324 D. 376 E. 432

1. A
2. E
3. E
4. A
5. D
6. D
7. A
8. C
9. D
10. B
11. B
12. E
13. B
14. C
15. E
16. D
17. 1471
18. C
19. D
20. D

1. If P is the price of the item with tax, $1.08P$ would be the price paid after tax. The discount is $1 - P/(1.08P) = 1 - 1/1.08 \approx 7.4\%$ (Answer: A)

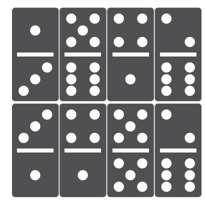
2. Rewrite each line in slope intercept form to find the slope of the first line is $m_1 = -\frac{a}{2}$ and the second is $m_2 = \frac{b}{3}$. If two lines are perpendicular, the product of their slopes is equal to $-1 \implies -\frac{a}{2} \cdot \frac{b}{3} = -1 \implies ab = 6$ (Answer: E)

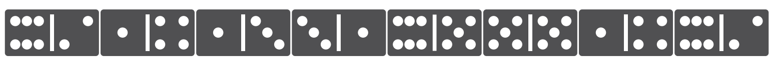
3. Sue's loan decreases by \$200 − \$10 = \$190 each month. $12000/190 \approx 63.2$. So after 63 months, she has paid her loan down to $12000 - 63 \times 190 = 30$. In the 64th month, she pays \$30 plus the \$10 in interest and the loan is paid off. (Answer: E)

4. $3x^2 + 4xy - 4y^2 = (3x - 2y)(x + 2y)$. $3x - 2y + x + 2y = 4x$ (Answer: A)

5. Solve the system to get the solution $\{(-3, 4)\}$. $a + b = 1$ (Answer: D)

6. The rectangle with the greatest perimeter is formed by placing all 8 dominos end-to-end resulting in a perimeter of 34. The rectangle with the smallest perimeter is a 4 by 4 square, with a perimeter of 16. $34/16 = 2.125$ (Answer: D)





7. The number must have 5 and 9 as a factor. Therefore the last digit must be either a 0 or a 5 and the sum of the digits must be divisible by 9. $(8, 0)$ and $(3, 5)$ both work and both have a sum of 8. (Answer: A)

8. $A_{Ed} + A_{Em} = 28$ and $\frac{1}{2}A_{Ed} = \frac{2}{3}A_{Em}$. Solve the system for just one of the variables and you have the solution. $A_{Ed} = 16$ and $A_{Em} = 12$. Both have 8 oz left. (Answer: C)

9. $2^{60} - 1$ is too big to test in a calculator but $2^{60} - 1 = (2^{30} - 1)(2^{30} + 1)$ and these two numbers are only 10 digits. Using a calculator you can determine $2^{30} - 1$, is divisible by 3, 7, and 13 and $2^{30} + 1$ is divisible by 5 and 11. 17 is the only number in the set that is not a factor. (Answer: D)

10. Let x be the number of 49¢ stamps and y be the number of 3¢ stamps. $0.49x + 0.03y = 4.10$. With $x > y$, there aren't very many combinations. Try the maximum value for x, which is 8 and it works! $x = 8$ and $y = 6$ (Answer: B)

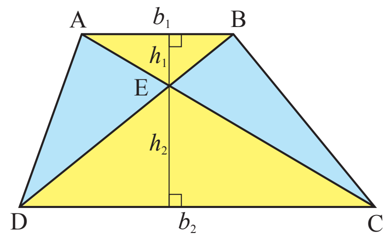
11. Evaluating $\sqrt[4]{2014}$ on your calculator will quickly give you the maximum possible value for a, which is 6. Using the TABLE function, quickly scan for integer values of $Y = \sqrt{2014 - 6^4 - X^2}$. You'll notice, no such values exist for $a = 6, 5, 4$, but for $a = 3$ we get $13 = \sqrt{2014 - 3^4 - 42^2}$ or $3^4 + 13^2 + 42^2 = 2014$ (Answer: B)

12. This can be done using trial and error. Die 1 = {B, A, C, D, I, N}, Die 2 = {O, P, H, R, X, G}, and Die 3 = {W, Y, T, L, E, S}. The only word that can be spelled is "won." (Answer: E)

13. With a graphing calculator, enter $Y_1 = X/(63 - X)$ and use the table to see if any values round to 0.455. None do, so try $Y_1 = X/(64 - X)$ and it works! The numbers are 20 and 40. (Answer: B)

14. Solve for x in both to get $\dfrac{15 - b}{a} = \dfrac{b - a}{15}$. Now solve for b to get $b = \dfrac{a^2 + 225}{a + 15}$. Use the table on a graphing calculator to find integer values. All possible solutions are for (a, b) are:
$\{(0, 15), (3, 13), (10, 13), (15, 15), (30, 25)\}$. (Answer: C)

15. Add the first equation to -3 times the second equation and to 3 times the third equation to get: $6r + 25s + 28t + 48u + 64v = 37$. (Answer: E)



16. The two triangles are similar and the ratio of their areas is $\frac{75}{48}$ or $\frac{25}{16}$. The ratio of the bases and heights of similar triangles is equal to the square root of the ratio of their areas. If b_1 and h_1 are the base and height of $\triangle$ABE, then $\frac{5}{4}b_1$ and $\frac{5}{4}h_1$ are the base and height of $\triangle$CDE. $A_{\text{trap ABCD}} = \frac{1}{2}(b_1 + \frac{5}{4}b_1)(h_1 + \frac{5}{4}h_1) = \frac{81}{16}(\frac{1}{2}b_1 h_1)$. $A_{\triangle \text{ABE}} = \frac{1}{2}b_1 h_1 \implies A_{\text{trap ABCD}} = \frac{81}{16}(48) = 243$. (Answer: D)

17. We know $7^3p + 7^2q + 7r + s = 9^3q + 9^2r + 9s + p \implies 680q + 74r + 8s - 342p = 0$. Now we need to find restrictions on the variables to make guess-and-check easier. First, p and q cannot equal zero because the numbers are 4 digits. No digit can be more than 6 because one of the numbers is in base 7. We know q cannot be bigger than 3 because $9^3 \cdot 4 = 2187$ requires five digits in base 7. Suppose $q = 3$, p would have to equal 6 since if $p = 5$ it would not be big enough to bring the equation back to zero. But no combination of r and s work with $p = 3$ and $q = 6$. Similarly, if $p = 2$ we only need to try 4, 5, or 6 for q. The solution is $(q, r, s, p) = (2, 0, 1, 4)$. Which is what we should have tried in the first place since they always try to work the year into these exams! (Answer: 1471)

18. Think of this as distributing 5 chips in 7 bowls. 6 bowls represent the 6 candidates and the 7th represents a vote for no one. If you visualize any given scenario as 5 chips and 6 dividers that separate the chips, the problem is reduced to the number of ways to rearrange 11 things, where 5 are identical and 6 are identical: $\dfrac{11!}{5! \cdot 6!} = 462$ (Answer: C)

19. Counting multiplicity, we know P has 4 roots. If there are only two distinct roots, P can be written $P(x) = (x-a)^2(x-b)^2$ or $P(x) = (x-a)^3(x-b)$. Expanding the first one gives: $P(x) = x^4 - 2(a+b)x^3 + (a^2+b^2+4ab)x^2 - 2ab(a+b)x + a^2b^2$. Solve the system $a^2b^2 = 144$, $-2ab(a+b) = -24$ and you get the roots -4 and 3. So $P(x) = x^4 + 2x^3 - 23x^2 - 24x + 144$. (Answer: D)

20. Start by listing all possible odd-neighbored sets:

n	Sets	Number
	∅	1
1	∅, {1}	2
2	∅, {1}, {1, 2}	3
3	∅, {1}, {3}, {1, 3}, {1, 2, 3}	5
4	∅, {1}, {3}, {1, 3}, {3, 4}, {1, 2, 3}, {1, 3, 4}, {1, 2, 3, 4}	8
5	∅, {1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}, {1, 2, 3}, {1, 3, 5}, {3, 4, 5}, {1, 2, 3, 5}, {1, 3, 4, 5}, {1, 2, 3, 4, 5}	13

The Fibonacci sequence! When $n = 12$ the total number of sets would be 377, one of which is empty. (Answer: D)