17. For some positive constant $C$, a patient’s temperature change, $T$ due to a dose, $D$, of a drug is given by

$$T = \left(\frac{C}{2} - \frac{D}{3}\right)D^2$$

(a) What dosage maximizes the temperature change?

"... change, $T$, due to a dose, $D$, ..." tells us that the function is

$$T(D) = \left(\frac{C}{2} - \frac{D}{3}\right)D^2$$

and that the derivative is $\frac{dT}{dD}$.

Since we are differentiating with respect to $D$, we’ll distribute first:

$$T(D) = \frac{C}{2}D^2 - \frac{1}{3}D^3$$

To find the critical points, we solve $T'(D) = 0$, so

$$T'(D) = \frac{C}{2} \cdot 2D - \frac{1}{3} \cdot 3D^2 = 0$$

$$CD - D^2 = 0$$

$$D(C - D) = 0$$

So $D = 0$ or $C - D = 0 \Rightarrow D = C$

are the critical points.

Now the domain for this function is $D \geq 0$. We really cannot guess on a maximum value of $D$, since we just do not know enough about drug dosages.
Since $D=0$ is at the boundary of the domain, it is not a critical point. Note however, that

$$T(0) = \left( \frac{c}{2} - \frac{C}{3} \right) \cdot 0^2 = 0$$

So no dosage of the drug results in no temperature change.

For $D=C$, we apply SOT:

$$T''(D) = C - 2D$$

and

$$T''(C) = C - 2C = -C$$

Since $C > 0$, $-C < 0$, so the second derivative is negative, the function is concave down, and $D=C$ gives a local maximum.

Thus a dosage of $D=C$ maximizes the temperature change.
(b) The sensitivity of the body to the drug is defined as \( \frac{dT}{dD} \). What dosage maximizes sensitivity?

We want to find the maximum of \( \frac{dT}{dD} \) for \( D > 0 \).

To find the critical numbers of \( \frac{dT}{dD} \), we solve

\[
\frac{d^2T}{dD^2} = C - 2D = 0
\]

or \( D = \frac{C}{2} \).

Is this a maximum? The second derivative of \( \frac{dT}{dD} \) is

\[
\frac{d^3T}{dD^3} = -2 < 0
\]

and thus by SDT, the maximum sensitivity occurs at a dosage of \( D = \frac{C}{2} \).