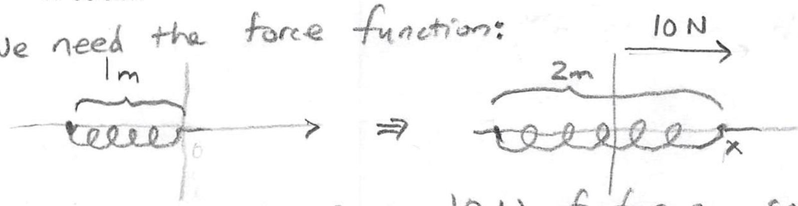
Math 185 (150B) name: ____________________

Sec. 6.4 Work date: ____________________

1. A spring has a natural length of 1 m, and a force of 10 N is required to hold it stretched to a total length of 2 m. How much work is done in compressing this spring from its natural length to a length of 60 cm?

We need the force function:

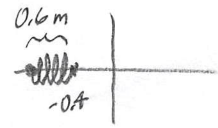


This stretch requires 10 N of force, so $10 = kx$. We've stretched the spring 1 m beyond its natural length, so $x = 1$.

$10 = k \cdot 1 \Rightarrow k = 10$.

The force function is $f(x) = 10x$.

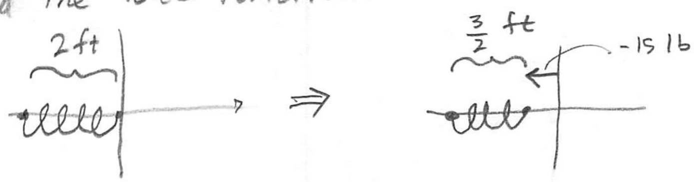
We compress the spring to a length of 0.6 m, so



$$\text{Work} = \int_{x=0}^{x=-0.4} 10x\,dx = \left[5x^2\right]_{x=0}^{x=-0.4} = \left(5(-0.4)^2 - 5(0)^2\right) = 0.8\text{ J}$$

2. A spring has a natural length of 2 ft, and a force of 15 lb is required to hold it compressed at a length of 18 in. How much work is done in stretching this spring from its natural length to a length of 3 ft?

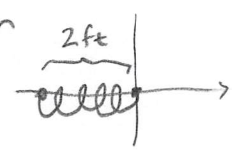
We need the force function:



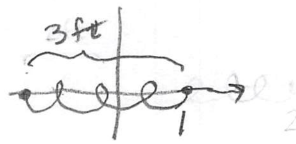
So $-15 = k\left(-\frac{1}{2}\right)$ or $k = 30$.

The force function is $f(x) = 30x$.

Now we stretch from



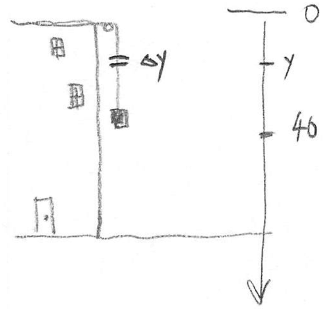
to



So

$$\text{Work} = \int_{x=0}^{x=1} 30x\,dx = \left[15x^2\right]_{x=0}^{x=1} = \left(15(1)^2 - 15(0)^2\right) = 15\text{ ft-lb}$$

3. A cable 40 m long with density 0.8 kg/m hangs from a 300 m tall building. A crate with mass of 200 kg is attached to the end of the cable. How much work is required to pull this cable and crate to the top of the building?



Let Δy = length of each segment of cable.

Then

$$\text{mass of segment} = 0.8\,\Delta y$$

$$\text{force on segment} = (9.8)(0.8\Delta y) = 7.84\,\Delta y$$

Since the segment travels a distance of y m

$$\text{Work on segment} = \underbrace{(7.84\Delta y)}_{\text{force}}\,\underbrace{y}_{\text{distance}}$$

Thus,

$$W_{cable} = \int_{y=0}^{y=40} 7.84\,y\,\Delta y = 7.84\left[\frac{y^2}{2}\right]_{y=0}^{y=40} = 7.84\left[\frac{(40)^2}{2} - \frac{(0)^2}{2}\right] = 6272\text{ J}$$

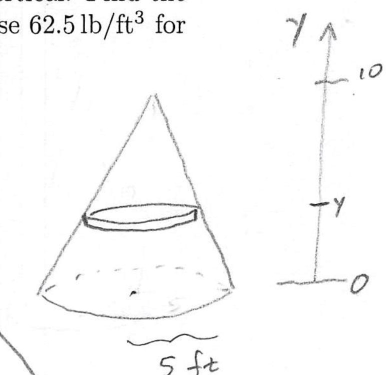
The crate moves the entire 40 m, so

$$W_{crate} = (200)(9.8)(40) = 78400\text{ J}$$

Finally,

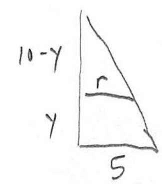
$$W_{total} = 6272\text{ J} + 78400\text{ J} = 84\,672\text{ J}$$

4. A conical tank of radius 5 ft and height 10 ft is resting on the ground with its axis vertical. Find the amount of work done in filling this tank with water pumped in from ground level. (Use 62.5 lb/ft^3 for the weight density of water.)



Since we are pumping in water from ground level (0 on the diagram), each slice must travel y ft

$$\text{volume of slice} = \pi\,(\text{radius of slice})^2\,\Delta y = \pi\left(\tfrac{1}{2}(10-y)\right)^2\Delta y = \frac{\pi}{4}(10-y)^2\Delta y$$



So $\dfrac{5}{10} = \dfrac{r}{10-y}$

$r = \frac{1}{2}(10-y)$

$$\text{force on slice} = \frac{\pi}{4}(10-y)^2\Delta y \cdot 62.5 = 15.625\pi\,(100 - 20y + y^2)\,\Delta y$$

$$\text{work on slice} = \underbrace{15.625\pi\,(100-20y+y^2)\Delta y}_{\text{force}} \cdot \underbrace{y}_{\text{distance}} = 15.625\pi\,(100y - 20y^2 + y^3)\,\Delta y$$

Since we move slices from $y=0$ to $y=10$,

$$W = \int_{y=0}^{y=10} 15.625\pi\,(100y - 20y^2 + y^3)\,dy = 15.625\pi\left[50y^2 - \frac{20}{3}y^3 + \frac{y^4}{4}\right]_{y=0}^{y=10}$$

$$= 15.625\pi\left[50(10)^2 - \frac{20}{3}(10)^3 + \frac{(10)^4}{4} - (0 - 0 + 0)\right] \approx 40{,}906.15\text{ ft-lb}$$