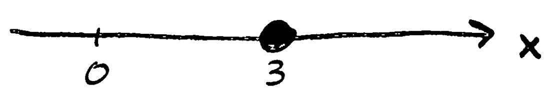
Math 280 (250A), Intermediate Calculus

Sec. 12.1 (10.1), Three-Dimensional Coordinate Systems

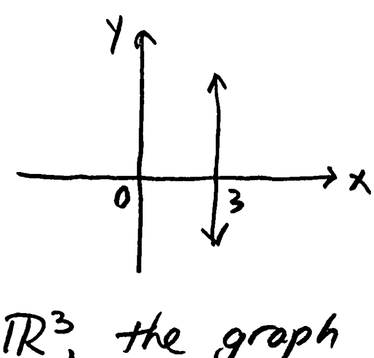
Convincing Sketches of Surfaces in $\mathbb{R}^3$

The graph of an equation depends on the space in which we are working. In previous courses, we have worked with one-dimensional spaces ($\mathbb{R}$, real numbers, or a single number line) and two-dimensional spaces ($\mathbb{R}^2$, or the familiar xy-coordinate system.)

Consider the equation $x=3$. In $\mathbb{R}$, the graph* of this equation is a single point on a number line



In $\mathbb{R}^2$, the graph of the equation $x=3$ consists of all the points on the xy-plane where the x-coordinate is three. We all know the graph is



In $\mathbb{R}^3$, the graph of the equation $x=3$ consists of all the points in three-dimensional space where the x-coordinate is three. To sketch a convincing

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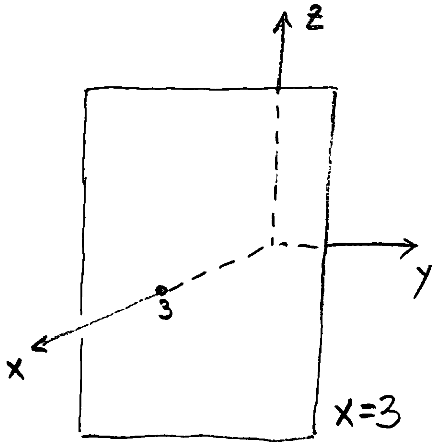
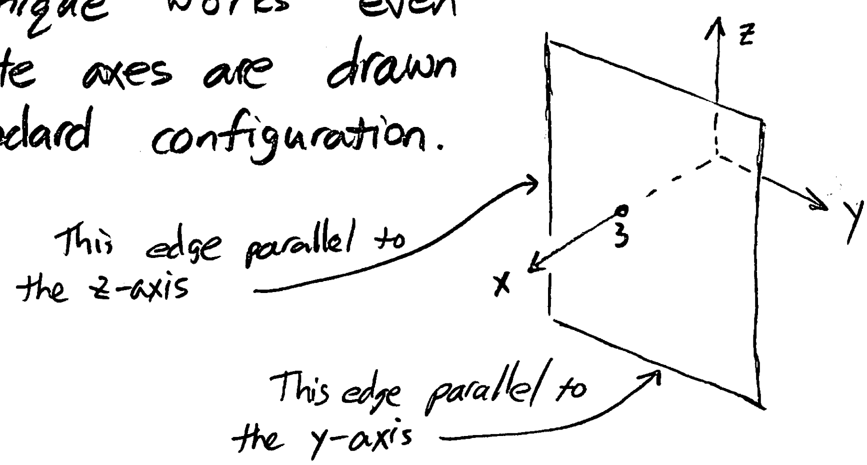
* A graph is a pictorial representation of the solutions to an equation or inequality.

graph of $x=3$ in $\mathbb{R}^3$, note that neither y nor z appear in the equation, so we draw the edges parallel to the y- and z-axes.

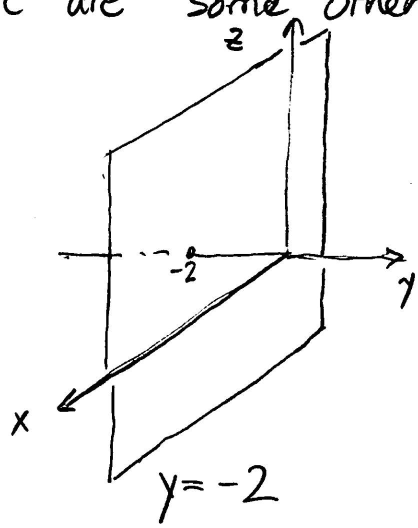


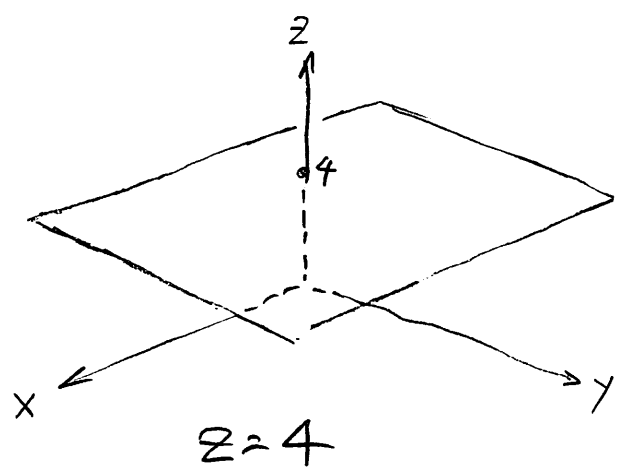
In $\mathbb{R}^3$, the graph of the equation $x=3$ is the set of all points of the form $(3, y, z)$.

This technique works even if the coordinate axes are drawn in a non-standard configuration.



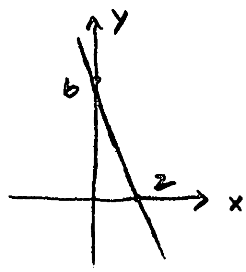
Here are some other sketches.



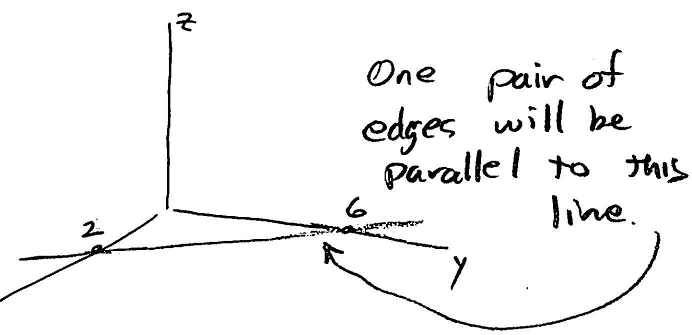


Let's sketch $2x + y = 6$ in $\mathbb{R}^3$.

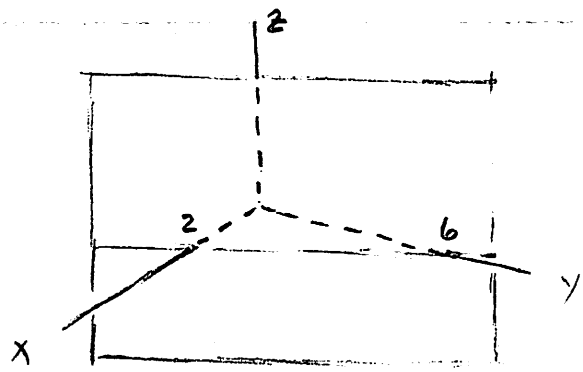
First, we'll sketch this in the xy-plane using what we learned way back in Algebra I.



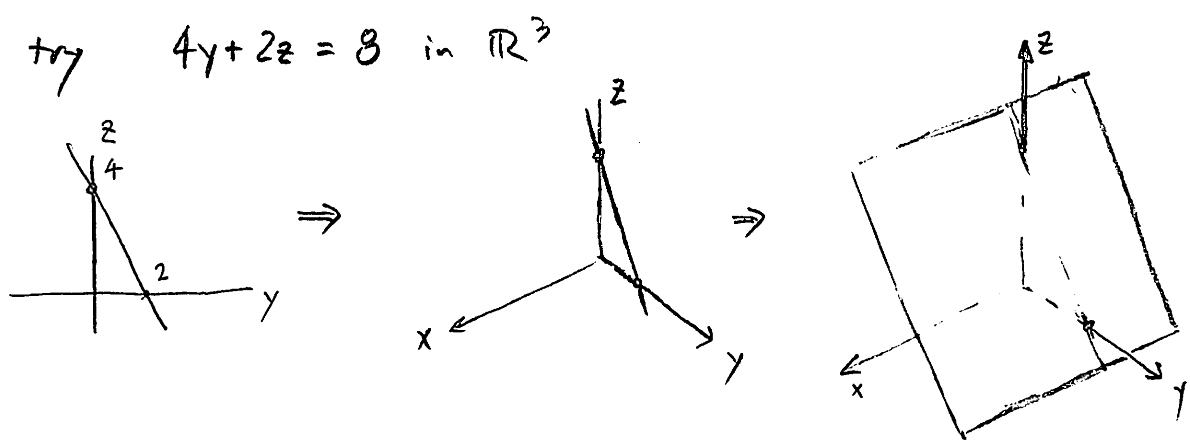
Now we sketch this line in $\mathbb{R}^3$.



Since z is missing from the equation the other pair of edges will be parallel to the z-axis.



Let's try $4y + 2z = 8$ in $\mathbb{R}^3$



As we progress through the rest of the course, we will learn how to draw many other surfaces.