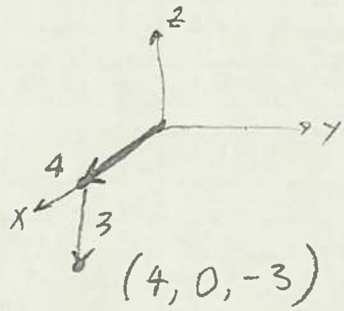
Sec. 12.1 #1, 3-6, 8, 11-13, 15, 17, 18, 25, 27, 29, 30

1. x-axis ... 4 units positive

downward 3 units

$(4, 0, -3)$

No movement in y direction



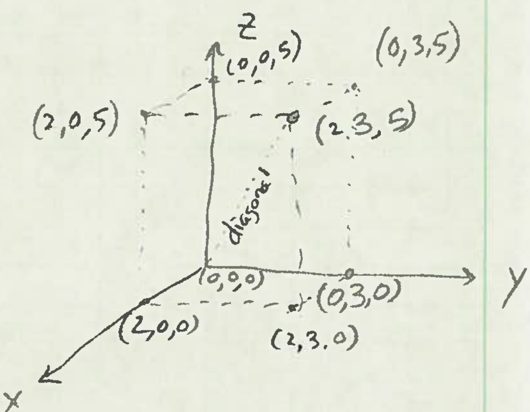
3. $A(-4,0,-1)$, $B(3,1,-5)$, $C(2,4,6)$

... Closest to yz-plane .. C is 2 units from yz-plane

... in xz-plane ... A has y-coordinate 0.

4. $(2,3,5)$

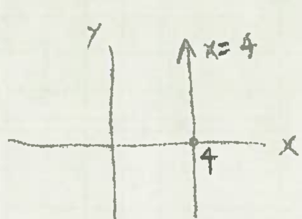
... onto xy-plane $(2,3,0)$
yz-plane $(0,3,5)$
xz-plane $(2,0,5)$



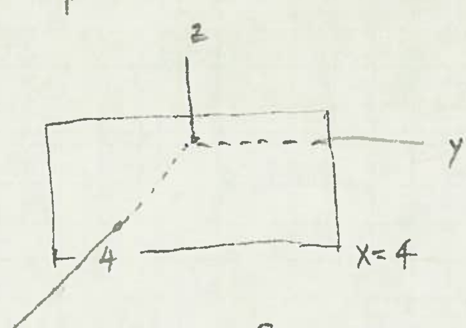
$$\text{diagonal} = \sqrt{(2-0)^2 + (3-0)^2 + (5-0)^2}$$
$$= \sqrt{4 + 9 + 25}$$
$$= \sqrt{38}$$

5. $x = 4$ ... In $\mathbb{R}^2$ is a line with all x-coordinates 4

.... in $\mathbb{R}^3$ is a plane with all x-coordinates 4



$\mathbb{R}^2$

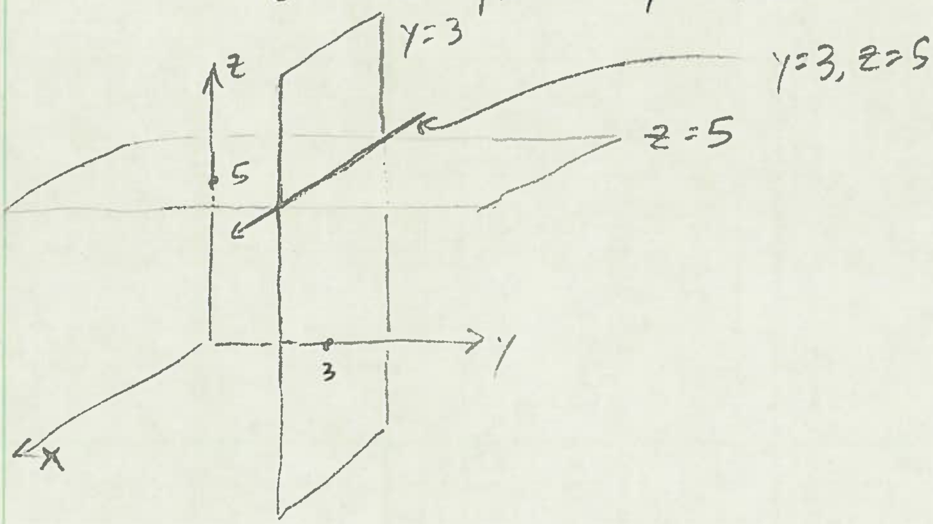


$\mathbb{R}^3$

6. $y=3$ ... in $\mathbb{R}^3$ is a plane where all y-coordinates are 3. The plane is parallel to the xz-plane.

$z=5$ .. in $\mathbb{R}^3$ is a plane where all z-coord's are 5. The plane is parallel to the xy-plane.

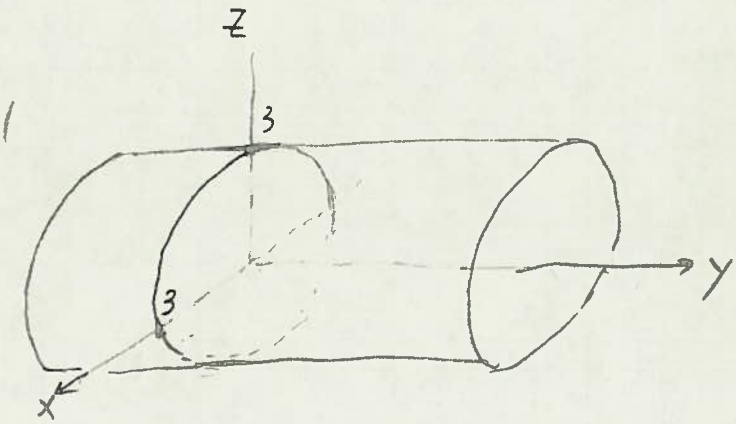
$y=3, z=5$... is a line where all y- an z-coord's are 3 and 5 respectively. The line is the intersection of the planes $y=3$ and $z=5$.



8. ... in $\mathbb{R}^3$, $x^2+z^2=9$

This is a cylinder, parallel to the y-axis, of radius 3.

($x^2+z^2=9$ is the trace of the cylinder in the xz-plane.)

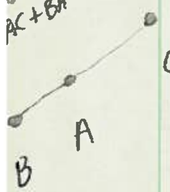


11. (a) $AB = \sqrt{(3-2)^2 + (7-4)^2 + (-2-2)^2} = \sqrt{1+9+16} = \sqrt{26}$

$BC = \sqrt{(1-3)^2 + (3-7)^2 + (3--2)^2} = \sqrt{4+16+25} = \sqrt{45}$

$AC = \sqrt{(1-2)^2 + (3-4)^2 + (3-2)^2} = \sqrt{1+1+1} = \sqrt{3}$

collinear if AC + BA = BC longest



But $\sqrt{3} + \sqrt{26} \neq \sqrt{45}$, so NOT on a straight line

(b) $DE = \sqrt{(1-0)^2 + (-2--5)^2 + (4-5)^2} = \sqrt{1+9+1} = \sqrt{11}$

$DF = \sqrt{(3-0)^2 + (4--5)^2 + (2-5)^2} = \sqrt{9+81+9} = \sqrt{99} = 3\sqrt{11}$

$EF = \sqrt{(3-1)^2 + (4--2)^2 + (2-4)^2} = \sqrt{4+36+4} = \sqrt{44} = 2\sqrt{11}$

But $\sqrt{11} + 2\sqrt{11} = 3\sqrt{11}$, so these points lie on a straight line.

12. $(4,-2,6)$ to

(a) xy-plane ... 6 (b) yz-plane ... 4 (c) xz-plane ... 2

(d) x-axis

$d = \sqrt{(4-4)^2 + (-2-0)^2 + (6-0)^2}$

$= \sqrt{0+4+36}$

$= \sqrt{40}$

or $= 2\sqrt{10}$

(e) y-axis

$d = \sqrt{(4-0)^2 + (-2--2)^2 + (6-0)^2}$

$= \sqrt{16+0+36}$

$= \sqrt{52}$

$= 2\sqrt{13}$

(f) z-axis

$d = \sqrt{(4-0)^2 + (-2-0)^2 + (6-6)^2}$

$= \sqrt{16+4+0}$

$= \sqrt{20}$

$= 2\sqrt{5}$

13. center $(-3,2,5)$, radius $= 4$

$$(x--3)^2 + (y-2)^2 + (z-5)^2 = 4^2$$

$$(x+3)^2 + (y-2)^2 + (z-5)^2 = 16$$

... intersection with yz-plane $\Rightarrow$ $x=0$

Substitute

$$(0+3)^2 + (y-2)^2 + (z-5)^2 = 16$$

$$9 + (y-2)^2 + (z-5)^2 = 16$$

$$(y-2)^2 + (z-5)^2 = 7$$

This is a circle in the yz-plane, centered at $(0,2,5)$ with radius $\sqrt{7}$

15. ∴ through $(4,3,-1)$, center $(3,8,1)$

$$\text{radius} = \sqrt{(4-3)^2+(3-8)^2+(-1-1)^2}$$
$$= \sqrt{1^2+(-5)^2+(-2)^2}$$
$$= \sqrt{30}$$

The equ'n of the sphere is $(x-3)^2+(y-8)^2+(z-1)^2 = 30$

17. $x^2+y^2+z^2-2x-4y+8z=15$

$$x^2-2x \quad +y^2-4y \quad +z^2+8z \quad = 15$$
$$x^2-2x+1 \quad +y^2-4y+4 \quad +z^2+8z+16 \quad = 15+1+4+16$$

$\frac{1}{2}(-2)$

-1

$(-1)^2=1$

$$(x-1)^2+(y-2)^2+(z+4)^2 = 36$$

This is a sphere with radius 6, centered at $(1,2,-4)$.

18. $x^2+y^2+z^2+8x-6y+2z+17=0$

$$x^2+8x \quad +y^2-6y \quad +z^2+2z \quad = -17$$
$$x^2+8x+16 \quad +y^2-6y+9 \quad +z^2+2z+1 \quad = -17+16+9+1$$
$$(x+4)^2 \quad +(y-3)^2 \quad +(z+1)^2 \quad = 9$$

this is a sphere with radius 3, centered at $(-4,3,-1)$.

25. $x=5$ is a plane, parallel to the yz-plane, with all x-coordinates 5.

27. $y<8$ is the part (half!) of $\mathbb{R}^3$ consisting of all points to the left of the plane $y=8$ (parallel to xz-plane).

29. $0 \le z \le 6$ is the part of $\mathbb{R}^3$ between the planes $z=0$ and $z=6$, both of which are parallel to the xy-plane

30. $y^2=4 \Rightarrow y^2-4=0 \Rightarrow (y+2)(y-2)=0 \Rightarrow y=-2$ or $y=2$ are the planes parallel to the xz-plane with y-coordinates -2 and 2, respectively.