Math 280 (250A), Intermediate Calculus
Sec. 12.1 (10.1), Three-Dimensional Coordinate Systems

**Convincing Sketches of Surfaces in \( \mathbb{R}^3 \)**

The graph of an equation depends on the space in which we are working. In previous courses, we have worked with one-dimensional spaces (\( \mathbb{R} \), real numbers, or a single number line) and two-dimensional spaces (\( \mathbb{R}^2 \), or the familiar xy-coordinate system.)

Consider the equation \( x = 3 \). In \( \mathbb{R} \), the graph of this equation is a single point on a number line.

\[ \begin{array}{c}
0 \quad 3 \\
\hline
x
\end{array} \]

In \( \mathbb{R}^2 \), the graph of the equation \( x = 3 \) consists of all the points on the xy-plane where the x-coordinate is three. We all know the graph is

\[ \begin{array}{c}
\uparrow \\
\hline
y
\end{array} \]

In \( \mathbb{R}^3 \), the graph of the equation \( x = 3 \) consists of all the points in three-dimensional space where the x-coordinate is three. To sketch a convincing

* A graph is a pictorial representation of the solutions to an equation or inequality.
graph of $x = 3$ in $\mathbb{R}^3$ note that neither $y$ nor $z$ appear in the equation, so we draw the edges parallel to the $y$- and $z$-axes.

In $\mathbb{R}^3$, the graph of the equation $x = 3$ is the set of all points of the form $(3, y, z)$.

This technique works even if the coordinate axes are drawn in a non-standard configuration.

Here are some other sketches.

$y = -2$

$z = 4$
Let's sketch $2x + y = 6$ in $\mathbb{R}^3$.

First, we'll sketch this in the $xy$-plane using what we learned way back in Algebra I.

Now we sketch this line in $\mathbb{R}^3$.

Since $z$ is missing from the equation the other pair of edges will be parallel to the $z$-axis.

Let's try $4y + 2z = 8$ in $\mathbb{R}^3$.

As we progress through the rest of the course, we will learn how to draw many other surfaces.