1. (a) $y = 5x^3 - 3x^2 + \sqrt[7]{x^3}$

$y = 5x^3 - 3x^2 + x^{3/7}$

so

$y' = 15x^2 - 6x + \frac{3}{7}x^{-4/7}$

$y' = 15x^2 - 6x + \frac{3}{7\sqrt[7]{x^4}}$

(b) $y = 3\sin(x) - \tan(x) + 2\sec(x)$

so $y' = 3\cos(x) - \sec^2(x) + 2\sec(x)\tan(x)$

2. (a) $y = (3x^2 - x^5)^4$

so

$y' = 4(3x^2 - x^5)^3(6x - 5x^4)$

$y' = 4x(6 - 5x^3)(3x^2 - x^5)^3$

$y' = 4x(6-5x^3)(x^2)^3(3-x^3)^3$

$y' = 4x^7(6-5x^3)(3-x^3)^3$

(b) $y = \sqrt{\arctan(x)}$

so

$y' = \frac{1}{2\sqrt{\arctan(x)}} \cdot \frac{1}{1+x^2}$

$y' = \frac{1}{2(1+x^2)\sqrt{\arctan(x)}}$

(c) $y = (\sin^{-1}(3x))^2$

so $y' = 2(\sin^{-1}(3x))^1 \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot 3$

$y' = \frac{6\sin^{-1}(3x)}{\sqrt{1-9x^2}}$

(d) $y = \sqrt{\cos(\sqrt{x})}$

so

$$y' = \frac{1}{2\sqrt{\cos(\sqrt{x})}} \cdot -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$y' = -\frac{\sin(\sqrt{x})}{4\sqrt{x}\sqrt{\cos(\sqrt{x})}}$$

$$y' = -\frac{\sin(\sqrt{x})}{4\sqrt{x\cos(\sqrt{x})}}$$

(e) $y = \tan\left(\frac{t}{1+t^2}\right)$

so

$$y' = \sec^2\left(\frac{t}{1+t^2}\right) \cdot \frac{(1+t^2)(1) - (t)(2t)}{(1+t^2)^2}$$

$$y' = \sec^2\left(\frac{t}{1+t^2}\right) \cdot \frac{-t^2+1}{(1+t^2)^2}$$

$$y' = \sec^2\left(\frac{t}{1+t^2}\right) \cdot \frac{1-t^2}{(1+t^2)^2}$$

3. (a) $y + x\cos(y) = x^2 y$

$$\frac{d}{dx}\left[y + x\cos(y)\right] = \frac{d}{dx}\left[x^2 y\right]$$

$$\frac{dy}{dx} + \frac{d}{dx}\left[x\cos(y)\right] = x^2 \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}\left[x^2\right]$$

$$\frac{dy}{dx} + x \cdot \frac{d}{dx}\left[\cos(y)\right] + \cos(y) \cdot \frac{d}{dx}\left[x\right] = x^2\frac{dy}{dx} + y \cdot 2x$$

$$\frac{dy}{dx} + x \cdot -\sin(y) \cdot \frac{dy}{dx} + \cos(y) \cdot 1 = x^2\frac{dy}{dx} + 2xy$$

$$\frac{dy}{dx} - x\sin(y)\frac{dy}{dx} - x^2\frac{dy}{dx} = 2xy - \cos(y)$$

$\longrightarrow$

$$\frac{dy}{dx}\left(1 - x\sin(y) - x^2\right) = 2xy - \cos(y)$$

$$\frac{dy}{dx} = \frac{2xy - \cos(y)}{1 - x\sin(y) - x^2}$$

(b) $\sin(xy) = x^2 - y$

$$\frac{d}{dx}\left[\sin(xy)\right] = \frac{d}{dx}\left[x^2 - y\right]$$

$$\cos(xy)\cdot\frac{d}{dx}\left[xy\right] = \frac{d}{dx}\left[x^2\right] - \frac{d}{dx}\left[y\right]$$

$$\cos(xy)\cdot\left(x\cdot\frac{d}{dx}[y] + y\cdot\frac{d}{dx}[x]\right) = 2x - \frac{dy}{dx}$$

$$\cos(xy)\left(x\frac{dy}{dx} + y\cdot 1\right) = 2x - \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = 2x - \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + \frac{dy}{dx} = 2x - y\cos(xy)$$

$$\frac{dy}{dx}\cdot\left(x\cos(xy) + 1\right) = 2x - y\cos(xy)$$

$$\frac{dy}{dx} = \frac{2x - y\cos(xy)}{x\cos(xy) + 1}$$

4. (a) $\lim_{x\to 0} \dfrac{x-\sin(x)}{x-\tan(x)} \xrightarrow[\text{Substitution}]{\text{Direct}} \dfrac{0-\sin(0)}{0-\tan(0)} \Rightarrow \dfrac{0}{0}$ indeterminate

$\overset{\text{L.H.}}{=} \lim_{x\to 0} \dfrac{1-\cos(x)}{1-\sec^2(x)} \overset{\text{D.S.}}{\Longrightarrow} \dfrac{1-\cos(0)}{1-\sec^2(0)} \Rightarrow \dfrac{1-1}{1-1} \Rightarrow \dfrac{0}{0}$ indeterminate

$\overset{\text{L.H.}}{=} \lim_{x\to 0} \dfrac{\sin(x)}{-2\sec(x)\cdot\sec(x)\tan(x)} \overset{\text{D.S.}}{\Rightarrow} \dfrac{\sin(0)}{-2\sec^2(0)\tan(0)} \Rightarrow \dfrac{0}{0}$ indeterminate

$\overset{\text{L.H.}}{=} \lim_{x\to 0} \dfrac{\cos(x)}{-2\left(\sec^2(x)\cdot\sec^2(x) + \tan(x)\cdot 2\sec(x)\sec(x)\tan(x)\right)}$

$\overset{\text{D.S.}}{\Longrightarrow} \dfrac{\cos(0)}{-2\left(\sec^4(0) + 2\sec^2(0)\tan^2(0)\right)}$

$= \dfrac{1}{-2(1+0)}$

$= -\dfrac{1}{2}$

(b) $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2x}{x-\sin(x)} \overset{\text{DS}}{\Longrightarrow} \dfrac{1-1-0}{0-0} = \dfrac{0}{0}$ indeterminate

$\overset{\text{L.H.}}{=} \lim_{x\to 0} \dfrac{e^x + e^{-x} - 2}{1-\cos(x)} \overset{\text{DS}}{\Longrightarrow} \dfrac{1+1-2}{1-1} = \dfrac{0}{0}$ indeterminate

$\overset{\text{L.H.}}{=} \lim_{x\to 0} \dfrac{e^x - e^{-x}}{\sin(x)} \overset{\text{DS}}{\Longrightarrow} \dfrac{1-1}{0} = \dfrac{0}{0}$ indeterminate

$\overset{\text{L.H}}{=} \lim_{x\to 0} \dfrac{e^x + e^{-x}}{\cos(x)}$

$\overset{\text{DS}}{\Longrightarrow} \dfrac{1+1}{1}$

$= 2$

5. (a) $\int \sqrt{x^3} + \sqrt[3]{x^2}\, dx$

$= \int x^{3/2} + x^{2/3}\, dx$

$= \frac{x^{3/2+1}}{3/2+1} + \frac{x^{2/3+1}}{2/3+1} + C$

$= \frac{2}{5}x^{5/2} + \frac{3}{5}x^{5/3} + C$

$= \frac{2}{5}\sqrt{x^5} + \frac{3}{5}\sqrt[3]{x^5} + C$

(b) $\int \sec(t)\left(\sec(t) + \tan(t)\right) dt$

$= \int \sec^2(t) + \sec(t)\tan(t)\, dt$

$= \tan(t) + \sec(t) + C$

6. (a) $\int_{x=1}^{x=2} \frac{1}{x^2} - \frac{4}{x^3}\, dx$

$= \int_{x=1}^{x=2} x^{-2} - 4x^{-3} dx$

$= \left[\frac{x^{-1}}{-1} - \frac{4x^{-2}}{-2} \right]_{x=1}^{x=2}$

$= \left[-\frac{1}{x} + \frac{2}{x^2} \right]_{x=1}^{x=2}$

$= \left(-\frac{1}{2} + \frac{2}{2^2}\right) - \left(-\frac{1}{1} + \frac{2}{1^2}\right)$

$= (\ 0\ ) - (-1+2)$

$= -1$

(b) $$\int_{x=0}^{x=\frac{\pi}{4}} \frac{1+\cos^2(x)}{\cos^2(x)}\,dx$$

$$= \int_{x=0}^{x=\frac{\pi}{4}} \frac{1}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)}\,dx$$

$$= \int_{x=0}^{x=\pi/4} \sec^2(x) + 1\,dx$$

$$= \Big[\tan(x) + x\Big]_{x=0}^{x=\frac{\pi}{4}}$$

$$= \left(\tan\left(\tfrac{\pi}{4}\right) + \tfrac{\pi}{4}\right) - \left(\tan(0) + 0\right)$$

$$= \left(1 + \tfrac{\pi}{4}\right) - (0+0)$$

$$= \frac{4+\pi}{4}$$

7. (a) $\int \cos^4(\theta)\sin(\theta)\,d\theta$

Let $u = \cos(\theta)$, so $du = -\sin(\theta)\,d\theta$.

$$-1\cdot\int \cos^4(\theta)\cdot -\sin(\theta)\,d\theta$$

$$= -\int u^4\,du$$

$$= -\left(\frac{u^5}{5} + C\right)$$

$$= -\left(\frac{\cos^5(\theta)}{5} + C\right)$$

$$= -\frac{1}{5}\cos^5(\theta) \underbrace{-C}_{\text{Just a constant, which we write as}}$$

$$= -\frac{1}{5}\cos^5(\theta) + C$$

(b) $\displaystyle\int_{x=0}^{x=1} x e^{-x^2} dx$

Let $u = -x^2$, so $du = -2x\,dx$.

When $x=0$, $u = -(0)^2 = 0$ and when $x=1$, $u = -(1)^2 = -1$.

$$-\frac{1}{2}\int_{x=0}^{x=1} -2x\, e^{-x^2} dx$$

$$= -\frac{1}{2}\int_{u=0}^{u=-1} e^u\, du$$

$$= -\frac{1}{2}\Big[e^u\Big]_{u=0}^{u=-1}$$

$$= -\frac{1}{2}\left[e^{-1} - e^0\right]$$

$$= -\frac{1}{2}\left[\frac{1}{e} - 1\right]$$

$$= -\frac{1}{2}\left(\frac{1-e}{e}\right)$$

$$= \frac{e-1}{2e}$$

8. (a) $\displaystyle\int \ln(x)\, dx$

Let $u = \ln(x)$, $dv = dx$,

so $du = \frac{1}{x}dx$, $v = x$.

$$\int \ln(x)dx = \ln(x)\cdot x - \int x \cdot \frac{1}{x} dx$$

$$= x\ln(x) - \int dx$$

$$= x\ln(x) - (x + C)$$

Just a constant, so we write

$$= x\ln(x) - x + C$$

(b) $\int t e^{-3t}\, dt$

$$\begin{array}{ccc} & u & dv \\ \hline + & t & e^{-3t} \\ - & 1 & -\frac{1}{3}e^{-3t} \\ + & 0 & \frac{1}{9}e^{-3t} \end{array}$$

$= + t \cdot -\frac{1}{3}e^{-3t} - 1 \cdot \frac{1}{9}e^{-3t} + \int 0 \cdot \frac{1}{9}e^{-3t}\, dt$

$= -\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + \int 0\, dt$

$= -\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C$

9. (a) $\int \cos^3(\theta)\sin^2(\theta)\, d\theta$

$= \int \cos^2(\theta)\sin^2(\theta)\, \underbrace{\cos(\theta)d\theta}$

If $u = \sin(\theta)$, then this will be part of du, so we rewrite the other factors in terms of $\sin(\theta)$.

$= \int (1-\sin^2(\theta))\sin^2(\theta)\, \cos(\theta)\, d\theta$

Let $u = \sin(\theta)$, so $du = \cos(\theta)d\theta$

$= \int (1-u^2)u^2\, du$

$= \int u^2 - u^4\, du$

$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= \frac{1}{3}\sin^3(\theta) - \frac{1}{5}\sin^5(\theta) + C$

(b) $\int \tan^2(x)\,dx$ $\quad 1+\tan^2(x) = \sec^2(x)$

$= \int \sec^2(x) - 1 \; dx$

$= \tan(x) - x + C$

10. (a) $\int \sqrt{1-4x^2}\,dx$

$= \int \sqrt{1-(2x)^2}\,dx$

Let $u = 2x$, $du = 2dx$.

$= \frac{1}{2}\int \sqrt{1-(2x)^2} \cdot 2\,dx$

$= \frac{1}{2}\int \sqrt{1-u^2}\,du$

Let $u = \sin(\theta)$, so $du = \cos(\theta)\,d\theta$

$= \frac{1}{2}\int \sqrt{1-\sin^2(\theta)} \cdot \cos(\theta)\,d\theta$

$= \frac{1}{2}\int \sqrt{\cos^2(\theta)} \cdot \cos(\theta)\,d\theta$

$= \frac{1}{2}\int |\cos(\theta)| \cdot \cos(\theta)\,d\theta$

If $\theta \in$ Quad I or IV, then $|\cos(\theta)| = \cos(\theta)$. Thus,

$= \frac{1}{2}\int \cos(\theta) \cdot \cos(\theta)\,d\theta$

$= \frac{1}{2}\int \cos^2(\theta)\,d\theta$

$= \frac{1}{2}\int \frac{1}{2} + \frac{1}{2}\cos(2\theta)\,d\theta$

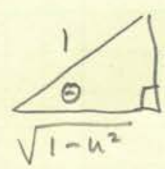
$= \frac{1}{2}\left[\frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C\right]$

Just a constant

$= \frac{1}{4}\theta + \frac{1}{8}\sin(2\theta) + C$

$= \frac{1}{4}\theta + \frac{1}{8}\cdot 2\cos(\theta)\sin(\theta) + C$

Since $u = \sin(\theta)$, we have



$\cos(\theta) = \sqrt{1-u^2}$

and $\theta = \sin^{-1}(u)$, so

$= \frac{1}{4}\sin^{-1}(u) + \frac{1}{4}\sqrt{1-u^2} \cdot u + C$

Since $u = 2x$, we have

$= \frac{1}{4}\sin^{-1}(2x) + \frac{1}{4}\sqrt{1-(2x)^2} \cdot 2x + C$

$= \frac{1}{4}\sin^{-1}(2x) + \frac{1}{2}x\sqrt{1-4x^2} + C$

(b) $\displaystyle\int_{x=0}^{x=3} \frac{x}{\sqrt{36-x^2}}\,dx$

As a trig. substitution ...

$$= \int_{x=0}^{x=3} \frac{x}{\sqrt{36\left(1-\frac{x^2}{36}\right)}}\,dx$$

$$= \int_{x=0}^{x=3} \frac{x}{6\sqrt{1-\left(\frac{x}{6}\right)^2}}\,dx$$

Let $u = \frac{x}{6}$, so $du = \frac{1}{6}\,dx$

and $6u = x$. When $x=0$, $u = \frac{0}{6} = 0$

and when $x=3$, $u = \frac{3}{6} = \frac{1}{2}$.

$$= \int_{u=0}^{u=\frac{1}{2}} \frac{6u}{\sqrt{1-u^2}}\,du$$

Let $u = \sin(\theta)$, so $du = \cos(\theta)\,d\theta$.

When $u=0$, we have $0 = \sin(\theta) \Rightarrow \theta = 0$

When $u = \frac{1}{2}$, we have $\frac{1}{2} = \sin(\theta) \Rightarrow \theta = \frac{\pi}{6}$

$$= \int_{\theta=0}^{\theta=\frac{\pi}{6}} \frac{6\sin(\theta)}{\sqrt{1-\sin^2(\theta)}}\cos(\theta)\,d\theta$$

$$= \int_{\theta=0}^{\theta=\frac{\pi}{6}} \frac{6\sin(\theta)}{\sqrt{\cos^2(\theta)}}\cos(\theta)\,d\theta$$

If $\theta \in$ Quad. I or $\underline{IV}$, $\sqrt{\cos^2(\theta)} = \cos(\theta)$.

$$= \int_{\theta=0}^{\theta=\pi/6} \frac{6\sin(\theta)}{\cos(\theta)}\cos(\theta)\,d\theta$$

$$= \int_{\theta=0}^{\theta=\pi/6} 6\sin(\theta)\,d\theta$$

$$= \Big[-6\cos(\theta)\Big]_{\theta=0}^{\theta=\pi/6}$$

$$= -6\cos\left(\tfrac{\pi}{6}\right) - -6\cos(0)$$

$$= -6\cdot\frac{\sqrt{3}}{2} + 6\cdot 1$$

$$= 6\left(1-\frac{\sqrt{3}}{2}\right)$$

$$= 6\left(\frac{2-\sqrt{3}}{2}\right)$$

$$= 3\left(2-\sqrt{3}\right)$$

As a u-sub...

Let $u = 36 - x^2$, $du = -2x\,dx$.

When $x=0$, $u=36$ and when $x=3$, $u = 36-9 = 27$.

$$-\frac{1}{2}\int_{x=0}^{x=3} \frac{-2x}{\sqrt{36-x^2}}\,dx$$

$$= -\frac{1}{2}\int_{u=36}^{u=27} \frac{1}{\sqrt{u}}\,du$$

$$= -\frac{1}{2}\int_{u=36}^{u=27} u^{-1/2}\,du$$

$$= -\frac{1}{2}\Big[2u^{1/2}\Big]_{u=36}^{u=27}$$

$$= -\frac{1}{2}\left[2\sqrt{27} - 2\sqrt{36}\right]$$

$$= -\frac{1}{2}\left[2\cdot 3\sqrt{3} - 2\cdot 6\right]$$

$$= -\frac{1}{2}\left[6\sqrt{3} - 12\right]$$

$$= -3\sqrt{3} + 6$$

$$= 6 - 3\sqrt{3}$$

$$= 3\left(2-\sqrt{3}\right)$$

11. (a) $\int \frac{10}{(x-1)(x^2+9)}\,dx$

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$10 = A(x^2+9) + (Bx+C)(x-1)$$

$$10 = Ax^2 + 9A + Bx^2 - Bx + Cx - C$$

$$10 = Ax^2 + Bx^2 - Bx + Cx + 9A - C$$

$$10 = (A+B)x^2 + (-B+C)x + (9A-C)$$

So

① $A + B = 0$

② $-B + C = 0$

③ $9A - C = 10$

If we add all three, we get

$$10A = 10$$

$$A = 1$$

From ①, we get $B = -1$ and from ② we get $C = -1$.

So

$$\frac{10}{(x-1)(x^2+9)} = \frac{1}{x-1} + \frac{-x-1}{x^2+9}$$

and

$$\int \frac{10}{(x-1)(x^2+9)}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{-x-1}{x^2+9}\,dx$$

$$= \int \frac{1}{x-1}\,dx + \int \frac{-x}{x^2+9}\,dx + \int \frac{-1}{x^2+9}\,dx$$

Let $u = x^2+9$, $du = 2x\,dx$

$$= \ln|x-1| + -\frac{1}{2}\int \frac{2x}{x^2+9}\,dx - \int \frac{1}{x^2+(3)^2}\,dx$$

$$= \ln|x-1| - \frac{1}{2}\int \frac{1}{u}\,du - \frac{1}{3}\arctan\left(\frac{x}{3}\right)$$

$$= \ln|x-1| - \frac{1}{2}\ln(x^2+9) - \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

(b) $\displaystyle\int \frac{5x+1}{2x^2-x-1}\,dx$

$$= \int \frac{5x+1}{(2x+1)(x-1)}\,dx$$

$$\frac{5x+1}{(2x+1)(x-1)} = \frac{A}{2x+1} + \frac{B}{x-1}$$

$$5x+1 = A(x-1) + B(2x+1)$$

$$5x+1 = Ax - A + 2Bx + B$$

$$5x+1 = (A+2B)x + (-A+B)$$

So

① $A + 2B = 5$

② $-A + B = 1$

① + ② $\Rightarrow \quad 3B = 6 \Rightarrow B = 2$

② $\Rightarrow \quad -A + 2 = 1 \Rightarrow -A = -1 \Rightarrow A = 1$

$$\frac{5x+1}{(2x+1)(x-1)} = \frac{1}{2x+1} + \frac{2}{x-1}$$

So,

$$\int \frac{5x+1}{(2x+1)(x-1)}\,dx = \int \frac{1}{2x+1}\,dx + \int \frac{2}{x-1}\,dx$$

$$= \frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C$$

Just a constant, so we can write

$$= \ln\left(\sqrt{2x+1}\right) + \ln\left((x-1)^2\right) + \ln(C)$$

$$= \ln\left[\sqrt{2x+1}\cdot(x-1)^2\cdot C\right]$$

$$= \ln\left[C\,(x-1)^2\sqrt{2x+1}\right]$$