CHAPTER 11: RATIONAL EQUATIONS AND APPLICATIONS

Chapter Objectives

By the end of this chapter, students should be able to:

✓ Identify extraneous values
✓ Apply methods of solving rational equations to solve rational equations
✓ Solve applications with rational equations including revenue, distance, and work-rate problems

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SECTION 11.1: RATIONAL EQUATIONS
When solving rational equations, we can solve by using the same strategy we used to solve linear equations with fractions: *clearing denominators*. However, we first need to revisit excluded values.

A. EXCLUDED VALUES REVIEW

**Note**
A rational expression is undefined where the denominator is zero.

**MEDIA LESSON**
Find excluded values of a rational expression (Duration 4:20)

*View the video lesson, take notes and complete the problems below*

Find values where a rational expression is undefined.

a) \[ \frac{17}{4x-32} \]  
b) \[ \frac{x+19}{(x-10)(x-4)} \]  
c) \[ \frac{9x+10}{x^2-9x-10} \]
Definition

Recall, the excluded values are values which make the expression undefined. Hence, when solving a rational equation, the solution(s) is any value(s) except the excluded values. If we obtain a solution that is an excluded value, we call this an extraneous solution.

B. SOLVE RATIONAL EQUATIONS BY CLEARING DENOMINATORS WITH THE LCD

Steps for solving rational equations with the same denominator

Step 1. Determine the excluded values of the equation.
Step 2. Clear denominators by multiplying each term by the lowest common denominator.
Step 3. Solve the equation.
Step 4. Verify that the solutions obtained are not an excluded value.

MEDIA LESSON

Solve rational equations by clearing the denominators-part 1 (Duration 9:15)

View the video lesson, take notes and complete the problems below

Example: Solve the rational equations.

a) \( \frac{x+8}{6} + 3 = \frac{3x+8}{8} \)

6: ______________________
8: ______________________
LCD: ___________________

Excluded values: ______________________

b) \( \frac{4}{x-2} + \frac{1}{2} = \frac{3}{4} \)

LCD= _____________________

You Try

a) Solve for \( x \): \( \frac{2}{3} x - \frac{5}{6} = \frac{3}{4} \)

b) Solve for \( x \): \( \frac{9}{x+1} - \frac{5}{2} = \frac{4}{3} \)
View the video lesson, take notes and complete the problems below

Example: Solve the rational equations.

c) \( \frac{2}{5x} - 3 = \frac{4}{x} \) \hspace{1cm} x \neq ____  
d) \( 2x - \frac{16}{x} = 4 \) \hspace{1cm} x \neq ____

YOU TRY

a) Solve for \( x \): \( \frac{5x+5}{x+2} + 3x = \frac{x^2}{x+2} \)

b) Solve for \( x \): \( \frac{x}{x+2} + \frac{1}{x+1} = \frac{5}{(x+1)(x+2)} \)
C. FACTORING DENOMINATORS

Often we will need to factor denominators before finding the LCD.

MEDIA LESSON

Solve rational equations with factoring the denominators first - Part 1 (Duration 4:27)

View the video lesson, take notes and complete the problems below

Example: Solve the rational equations.

a) \(\frac{x}{x-2} + \frac{x}{x^2-4} = \frac{x+3}{x+2}\)

Excluded values: __________

MEDIA LESSON

Solve rational equations with factoring the denominators first – Part 2 (Duration 4:45)

View the video lesson, take notes and complete the problems below

Example: Solve the rational equations.

b) \(\frac{x-3}{x+6} + \frac{x-2}{x-3} = \frac{x^2}{x^2+3x-18}\)

Excluded values: __________
YOU TRY

a) Solve for $t$: \[ \frac{t}{t-1} - \frac{1}{t-2} = \frac{11}{t^2-3t+2} \]

b) Solve for $x$: \[ \frac{2x}{x+1} + \frac{3x}{x+1} = \frac{x^2}{x^2+2x+1} \]

D. SOLVING RATIONAL EQUATIONS WITH EXTRANEOUS SOLUTIONS

 MEDIA LESSON
Solve a rational equation with no solution (Duration 5:07)

View the video lesson, take notes and complete the problems below

Solve the rational equations.

a) \[ \frac{x}{6x-36} - 9 = \frac{1}{x-6} \] 

Excluded values: __________
View the video lesson, take notes and complete the problems below

Rational equations – extraneous
Because we are working with fractions, the _______________ cannot be __________.

Solve the rational equations.

\[ \frac{x}{x-8} - \frac{2}{x-4} = \frac{-3x+56}{x^2-12x+32} \]

\[ \frac{x}{x-2} + \frac{2}{x-4} = \frac{4x-12}{x^2-6x+8} \]

YOU TRY

a) Solve for \( n \):
\[ \frac{n}{n+5} - \frac{2}{n-9} = \frac{-11n+15}{n^2-4n-45} \]
EXERCISE
Solve. Be sure to verify all solutions.

1) \(3x - \frac{1}{2} - \frac{1}{x} = 0\)

3) \(x + \frac{6}{x-3} = \frac{2x}{x-3}\)

5) \(\frac{3m}{2m-5} - \frac{7}{3m+1} = \frac{3}{2}\)

7) \(\frac{7}{3-x} + \frac{1}{2} = \frac{3}{4-x}\)

9) \(x + 1 = \frac{4}{x+1}\)

11) \(\frac{4x}{2x-6} - \frac{4}{5x-15} = \frac{1}{2}\)

13) \(\frac{x-2}{x+3} - \frac{1}{x-2} = \frac{1}{x^2+x-6}\)

15) \(\frac{x-5}{x-9} + \frac{x+3}{x-3} = \frac{-4x^2}{x^2-12x+27}\)

17) \(\frac{2x}{x+2} + \frac{2}{x-4} = \frac{3x}{x^2-2x-8}\)

19) \(\frac{6x+5}{2x^2-2x} + \frac{2}{x^2-1} = \frac{3x}{x^2-1}\)

2) \(x + \frac{20}{x-4} = \frac{5x}{x-4} - 2\)

4) \(\frac{2x}{3x-4} = \frac{4x+5}{6x-1} - \frac{3}{3x-4}\)

6) \(\frac{4-x}{1-x} = \frac{12}{3-x}\)

8) \(\frac{2}{3-x} - \frac{6}{8-x} = 1\)

10) \(\frac{x-4}{x-1} = \frac{12}{x-3} + 1\)

12) \(\frac{x-1}{x-3} + \frac{x+2}{x+3} = \frac{3}{4}\)

14) \(\frac{3x-5}{5x-5} + \frac{5x-1}{7x-7} - \frac{x-4}{1-x} = 2\)

16) \(\frac{2x}{x+1} - \frac{3}{x+5} = \frac{-8x^2}{x^2+6x+5}\)

18) \(\frac{x+2}{3x-1} - \frac{1}{x} = \frac{3x-3}{3x^2-x}\)

20) \(\frac{x-3}{x+6} + \frac{x-2}{x-3} = \frac{x^2}{x^2+3x-18}\)
SECTION 11.2: WORK-RATE PROBLEMS

Work-rate equation

If the first person does a job in time \( A \), a second person does a job in time \( B \), and together they can do a job in time \( T \) (total). We can use the work-rate equation:

\[
\frac{1}{A} + \frac{1}{B} = \frac{1}{T} \quad \text{job per time } A, \text{job per time } B, \text{job per time } T(\text{team})
\]

A. ONE UNKNOWN TIME

View the video lesson, take notes and complete the problems below

Adam does a job in 4 hours. Each hour he does ____________ of the job.
Betty does a job in 12 hours. Each hour she does ____________ of the job.
Together, each hour they do _____________________________ of the job.
This means it takes them, working together, _______ hours to do the entire job.

Work Equation: _____________________________________________ Use __________________!

Example 1: Catherine can paint a house in 15 hours. Dan can paint it in 30 hours. How long will it take them working together?

Catherine: _______ hours
Dan: _____________ hours
Team: ___________ hours

Example 2: Even can clean a room in 3 hours. If his sister Faith helps, it takes them \( 2 \frac{2}{5} \) hours. How long will it take Faith working alone?

Even: _________ hours
Faith: __________ hours
Team: ___________ hours
YOU TRY

a) If worker A can do a piece of work alone in 6 days and worker B can do it alone in 4 days, how long will it take the two working together to complete the job?

<table>
<thead>
<tr>
<th>Time</th>
<th>Job per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worker A</td>
<td></td>
</tr>
<tr>
<td>Worker B</td>
<td></td>
</tr>
<tr>
<td>Together</td>
<td></td>
</tr>
</tbody>
</table>

a) Adam can assemble a furniture set in 5 hours. If his sister Maria helps, they can finish it in 3 hours. How long will it take Maria to do the job alone?

<table>
<thead>
<tr>
<th>Time</th>
<th>Job per hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adam</td>
<td></td>
</tr>
<tr>
<td>Maria</td>
<td></td>
</tr>
<tr>
<td>Together</td>
<td></td>
</tr>
</tbody>
</table>

MEDIA LESSON
Fill and drain problem (Duration 0:54)

View the video lesson, take notes and complete the problems below

Example: One inlet pipe can fill an empty pool in 8 hours, and a drain can empty the pool in 12 hours. How long will it take the pipe to fill the pool if the drain is left open?

<table>
<thead>
<tr>
<th>Time</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet pipe</td>
<td></td>
</tr>
<tr>
<td>Drain</td>
<td></td>
</tr>
<tr>
<td>Together</td>
<td></td>
</tr>
</tbody>
</table>

Fill-drain equation:
YOU TRY

a) A sink can be filled by a pipe in 5 minutes, but it takes 7 minutes to drain a full sink. If both the pipe and the drain are opened, how long will it take to fill the sink?

<table>
<thead>
<tr>
<th>Time</th>
<th>Fill per minute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill the sink</td>
<td></td>
</tr>
<tr>
<td>Drain the sink</td>
<td></td>
</tr>
<tr>
<td>Together</td>
<td></td>
</tr>
</tbody>
</table>

B. TWO UNKNOWN TIMES

View the video lesson, take notes and complete the problems below

Example: If Alfonso does a job in 30 hours less than Zoe, and they can do the job together in 8 hours, long will it take each to do the job alone?

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Work rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alfonso</td>
<td></td>
</tr>
<tr>
<td>Zoe</td>
<td></td>
</tr>
<tr>
<td>Together</td>
<td></td>
</tr>
</tbody>
</table>
a) Mike takes twice as long as Rachel to complete a project. Together, they can complete a project in 10 hours. How long will it take each of them to complete a project alone?

<table>
<thead>
<tr>
<th>Time</th>
<th>Project/ hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mike</td>
<td></td>
</tr>
<tr>
<td>Rachel</td>
<td></td>
</tr>
<tr>
<td>Together</td>
<td></td>
</tr>
</tbody>
</table>

b) Brittney can build a large shed in 10 days less than Cosmo. If they built it together, it would take 12 days. How long would it take each of them working alone?

<table>
<thead>
<tr>
<th>Time</th>
<th>Built per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cosmo</td>
<td></td>
</tr>
<tr>
<td>Brittney</td>
<td></td>
</tr>
<tr>
<td>Together</td>
<td></td>
</tr>
</tbody>
</table>
EXERCISE

1) A tank can be filled by one pipe in 20 minutes and by another in 30 minutes. How long will it take both pipes together to fill the tank?

2) Tim can finish painting his barn in 10 hours. It takes his wife JoAnn only 8 hours to do the same job. If they work together, how long will it take them to complete the job?

3) Adan can do a piece of work in 3 days, Bernie in 4 days, and Cynthia in 6 days each working alone. How long will it take them to do it working together?

4) A carpenter and his assistant can do a piece of work in 3 days. If the carpenter himself could do the work alone in 5 days, how long would the assistant take to do the work alone?

5) A sink can be filled from the faucet in 5 minutes. It takes only 3 minutes to empty the sink when the drain is open. If the sink is full and both the faucet and the drain are open, how long will it take to empty the sink?

6) Of two inlet pipes, the smaller pipe takes 5 hours longer than the larger pipe to fill a pool. When both pipes are open, the pool is filled in 6 hours. If only the larger pipe is open, how many hours are required to fill the pool?

7) It takes John 16 minutes longer than Sally to mow the lawn. If they work together they can mow the lawn in 15 minutes. How long will it take each to mow the lawn if they work alone?

8) Bill’s father can paint a room in 3 hours less than Bill can paint it. Working together they can complete the job in 2 hours. How much time would each require working alone?

9) Two workers, a trainer and a trainee, working together can do a job in 3 hours. The trainer is 3 times faster than the trainee to complete the same job. How long will it take the trainee to finish the same job?

10) The faucet alone can fill the sink in 6 minutes, while it takes 8 minutes to empty it with the drain. How long will it take to fill the sink?

11) It takes Roberto 8 hours longer than Paula to repair a transmission. If it takes them 3 hours to do the job if they work together, how long will it take each of them working alone?

12) A water tank is being filled by two inlet pipes. Pipe A can fill the tank in 4 hours, while both pipes together can fill the tank in 2 hours. How long does it take to fill the tank using only Pipe B?

13) Cheng takes 10 hours longer to pave a driveway than Sammy to do a job. Working together they can do the job in 12 hours. How long does it take each working alone?
SECTION 11.3: UNIFORM MOTION PROBLEMS

We can recall uniform motion problems in the word problems chapter. We used the formula \( r \cdot t = d \) and organized the given information in a table. Now, we use the equation as:

\[
t = \frac{d}{r}
\]

We apply the same method in this section only the equations will be rational equations.

A. UNIFORM MOTION PROBLEMS

View the video lesson, take notes and complete the problems below

Caitlyn went on a 56 mile trip to a soccer game. On the way back, due to road construction she had to drive 28 miles per hour slower. This made the trip take 1 hour longer. How fast did she drive to the soccer game?

Recall:

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>t</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>To</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
a) Greg went to a conference in a city 120 miles away. On the way back, due to road construction he had to drive 10 mph slower which resulted in the return trip taking 2 hours longer. How fast did he drive on the way to the conference?
B. UNIFORM MOTION PROBLEMS WITH STREAMS AND WINDS

Another type of uniform motion problem is where a boat is traveling in a river with the current or against the current (or an airplane is flying with the wind or against the wind). If a boat is traveling downstream, the current will push it or increase the rate by the speed of the current. If a boat is traveling upstream, the current will pull against it or decrease the rate by the speed of the current.

| MEDIA LESSON | Uniform motion – Streams & winds – Part 1: set up the equation (Duration 4:28) |
| MEDIA LESSON | Uniform motion – Streams & winds – Part 2: solve the equation (Duration 5:55) |

View the video lesson, take notes and complete the problems below

Alicia rows a boat downstream for 182 miles. The return trip upstream took 12 hours longer. If the current flows at 3 mph, how fast does Alicia row in still water?

Recall: Uniform motion: ________________________________

Rational equation: ________________________________

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>t</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream (faster)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upstream (slower)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
YOU TRY

a) A man rows downstream for 30 miles then turns around and returns to his original location, the total trip took 8 hours. If the current flows at 2 miles per hour, how fast would the man row in still water?
EXERCISE

1) An athlete plans to row upstream a distance of 6 kilometers and then return to his starting point in a total time of 4 hours. If the rate of the current is 2 km/hr., how fast should he row?

2) A pilot flying at a constant rate against a headwind of 30 km/hr flew for 720 kilometers, then reversed direction and returned to his starting point. He completed the round trip in 10 hours. What was the speed of the plane?

3) Tim, an open water swimmer, is training for the Olympics. To do so, he swims in a stream that is 3m/h. Tim finds that he can swim 4 miles against the current in the same amount of time that he can swim 10 miles with the current. How fast can Tim swim with no current?

4) A plane flies against the wind 288 miles from LAX to San Jose and then returns home with the same wind. The wind speed is 60m/h. If the total flying time was 2 hours, what is the speed of the plane?

5) A salmon is swimming in a river that is flowing downstream at a speed of 2 miles per hour. The salmon can swim 12 miles upstream in the same amount of time it would take to swim 24 miles downstream. What is the speed of the salmon in still water?
SECTION 11.4: REVENUE PROBLEMS

Revenue problems are the problems where a person buys a certain number of items for a certain price per item. If we multiply the number of items by the price per item, we will get the total value. We can recall revenue problems in the word problems chapter. We used the formula \( AVT \):

\[
\text{Amount} \cdot \text{Value} = \text{Total} \\
\text{or} \quad n \cdot p = R
\]

View the video lesson, take notes and complete the problems below

<table>
<thead>
<tr>
<th>Revenue table:</th>
<th>Number of items</th>
<th>Price</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( n )</td>
<td>( p )</td>
<td>( R )</td>
</tr>
</tbody>
</table>

To solve: Divide by

Example 1: A group of college students bought a couch for $80. However, five of them failed to pay their share so the others had to each pay $8 more. How many students were in the original group?

\[
\begin{array}{ccc}
 n & p & R \\
\hline
 & & \\
\end{array}
\]

Example 2: A merchant bought several pieces of silk for $70. He sold all but two of them at a profit of $4 per piece. His total profit was $18. How many pieces did he originally purchase?

\[
\begin{array}{ccc}
 n & p & R \\
\hline
 & & \\
\end{array}
\]
YOU TRY

a) A man buys several fish for $56. After three fish die, he decides to sell the rest at a profit of $5 per fish. His total profit was $4. How many fish did he buy to begin with?

b) A group of students bought a couch for their dorm at cost $96. However, 2 students failed to pay their share, so each student had to pay $4 more. How many students were in the original group?
EXERCISE

1) A merchant bought some pieces of silk for $900. He marked up $15 more per each piece and made $75 profit. Find the number of pieces purchased.

2) A group of students planned to chip in to raise a total of $100 for one of their friends’ birthday party but 5 persons couldn’t come so they didn’t pay. This increased the share of the others by $1 each. Find the amount that each person paid after.

3) A merchant bought a number of barrels of apples for $120. He kept 2 barrels and sold the remainder at a profit of $2/barrel making a total profit of $34. How many barrels did he originally buy?

4) A dealer bought a number of sheep for $440. After 5 had died, he sold the remainder at a profit of $2 each, making a profit of $60 for the sheep. How many sheep did he originally purchase?

5) Orlando bought a number of hats at equal cost for $500. He sold all but 2 for $540 at a profit of $5 for each item. How many hats did he buy?

6) A fashion store bought a lot of suits for $750. The store sold all of them for $1,000 making a profit of $10 on each suit sold. How many suits did the store buy?

7) A group of schoolboys tried to raise $4,500 for a local animal shelter. Five boys transferred to different schools before they had a chance to raise money, hence each remaining boy was compelled to raise $45 more. How many boys were in the original group and how much had each agreed to raise?

8) The total expenses of a camping party were $72. If there had been 3 fewer persons in the party, it would have cost each person $2 more than it did. How many people were in the party and how much did it cost each one?
### KEY TERMS AND CONCEPTS

Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraneous solution</td>
<td></td>
</tr>
<tr>
<td>Work-rate equation</td>
<td></td>
</tr>
</tbody>
</table>